

Electromagnetic-wave Contribution to the Quantum Structure of Matter

Burke Ritchie

Livermore Software Technology Corporation
USA

1. Introduction

The quantum theory of matter does not describe real matter until electromagnetic theory is used to account for such diverse radiative phenomena as spontaneous emission and the shift of quantum energy levels. Classical electrodynamics fails to account quantitatively for these radiative effects in the structure of matter. Quantum electrodynamics (QED) does successfully account for radiative effects in the structure of matter once an infinite contribution to the energy, which diverges linearly with electromagnetic-wave frequency, is subtracted from the theory based on physical argument that such contribution is already included, to zeroth order in perturbation theory, in the description of a radiative as opposed to a nonradiative or bare electron. This mathematical procedure is known as mass renormalization and introduces the concept that total mass comprises both material and electromagnetic contributions, neither of which is observable by itself.

In Section II of this paper a theory is presented which describes both the material and radiative properties of matter in a single, inseparable form. We show that the time-domain relativistic-wave equation of Paul Dirac can be inferred from the Lorentz invariant obtained from the scalar product of the electron's four-momentum and an electromagnetic four-potential, once an electromagnetic carrier-wave frequency is formally identified with the

rest-mass energy of the electron divided by \hbar , namely $\omega = \frac{mc^2}{\hbar}$. (The scalar product of two

four-vectors always gives a Lorentz invariant such that the present derivation proves the Lorentz invariance of Dirac's equation in a single step. In the standard treatment [1], in which the Dirac Hamiltonian is the scalar product of two operator four-vectors, a second step is required to prove the Lorentz invariance of the wave equation itself.) Our derivation elucidates a long-studied problem in the literature of the identity of Dirac's equation with the spinorial form of Maxwell's equation [2-5]. The value of ω given above is just the cut off of the electromagnetic frequency used in QED to insure the finite value of the logarithmically-divergent contribution to the energy, which is the only divergent term remaining after the term linearly divergent in the frequency has been removed by mass renormalization. In summary Dirac's time-domain relativistic wave equation is reinterpreted to be an equation which accounts for both the material and radiative properties of matter.

In Section III we provide an analytic proof that Dirac's temporally harmonic solution of his equation is equivalent to solving temporally coupled equations by adiabatic elimination, which

is a widely-used approximation method to solve temporally coupled equations in the optical-physics literature. In a word Dirac's temporally harmonic solution is approximate, but his solution of the resulting time-independent or energy-domain equation, which is a staple of the relativistic quantum mechanics literature, is exact. The current interpretation of Dirac's theory as describing only the material properties of matter derives from Dirac's solution of his time-

domain equation using the harmonic substitution, $\Psi_D(\vec{r}, t) = e^{-i\frac{E}{\hbar}t} \Psi_{D,E}(\vec{r})$, albeit this form, since it constrains all four components of his energy-domain vector wave function $\Psi_{D,E}(\vec{r})$ to

oscillate in time at a single frequency $\omega = \frac{E}{\hbar}$, is obviously not the general solution. The same

harmonic form however exactly solves Schroedinger's time-domain equation and thus gives a result which is compatible with the way quantum theory evolved as a matter-only theory without radiative effects until augmented by QED.

In Section IV numerical results are presented for the general solution of Dirac's time-dependent equation. Fourier analysis of the general time-dependent solution shows that the spectrum of quantum states for the Coulomb problem comprises coupled positive- and negative-energy states. The wave function is a mixture of bound and continuum states, with an unbound component propagating away from the atom in a manner which satisfies the Lorentz-invariant relationship or causality between position and time, $r^{**2} - (ct)^{**2} = 0$. The unbound behavior has long been known as Zitterbewegung for a free electron, and here we show its counterpart for the Coulomb problem. In view of the Dirac-Maxwell relationship elucidated in Section II we postulate that the physical interpretation of Zitterbewegung is the emission of a photon with energy of order $2mc^{**2}$ due to the presence of empty negative-energy states in the general time-dependent solution. Dirac's artifice of filling up the negative-energy levels with electrons to stabilize the atom is not available in the general time-dependent solution.

In Section V equations of motion for the photon are given. In Section VI subatomic bound solutions are discovered which are expected due to the temporally second-order nature of the time-domain Dirac equation. Subatomic bound solutions do not exist for Dirac's time-independent equation (hereafter called standard Dirac theory) due to his use of the single-frequency temporally harmonic form discussed in Section III. Thus the existence of a complex neutron cannot be ruled out in the case of the general time-dependent solution as it was earlier in the case of standard Dirac theory [6]. These solutions resemble known Dirac energy-domain functions for $Z > 137$. The spectral content of these solutions comprises a spectral peak at $-mc^{**2}$ for an electron and a spectral peak at $+mc^{**2}$ for a positron - yes this state exists for a positive Coulomb potential. An electron can thus make an upward transition into the positive-energy continuum with transition energy $2mc^{**2}$, as in standard Dirac theory, while a positron can make a downward transition into the negative-energy continuum with transition energy $-2mc^{**2}$. The upward transition is considered to be a matter transition, while the downward transition is considered to be an anti-matter transition.

2. Maxwell-Dirac equivalency

There exists a physical equivalency between Dirac and Maxwell theories which can be stated as follows. It is well known that Lorentz' equation is the Lorentz invariant formed by taking the scalar product of the four-gradient and the electromagnetic four-potential,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot (\Phi, \vec{A}) = \frac{1}{c}\frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0 \quad (1)$$

Recall that the scalar product of four-vectors is always a Lorentz Invariant. One may postulate that a four-potential exists for the electron, such that an electron equation of motion can be written as the Lorentz invariant formed by taking the scalar product of the electron's four-momentum and the electron's four-potential,

$$\left(\frac{i\hbar}{c}\frac{\partial}{\partial t} - \frac{e}{c}\Phi, i\hbar\vec{\nabla} + e\vec{A}\right) \cdot (\Phi_e, \vec{A}_e) = \left(\frac{i\hbar}{c}\frac{\partial}{\partial t} - \frac{e}{c}\Phi\right)\Phi_e + (i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}) \cdot \vec{A}_e = 0 \quad (2)$$

The electron scalar and vector potentials can be written in the form of carrier-wave expansions,

$$\Phi_e = \Phi_{e-}e^{-i\omega_e t} + \Phi_{e+}e^{i\omega_e t}, \quad (3a)$$

$$\vec{A}_e = \vec{A}_{e-}e^{-i\omega_e t} + \vec{A}_{e+}e^{i\omega_e t}, \quad (3b)$$

from which on substituting Eqs. (3) into Eq. (2) and separately setting the coefficients of the exponential factors equal to zero, we obtain,

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi - \hbar\omega_e\right)\Phi_{e+} + (i\hbar c\vec{\nabla} + e\vec{A}) \cdot \vec{A}_{e+} = 0 \quad (4a)$$

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi + \hbar\omega_e\right)\Phi_{e-} + (i\hbar c\vec{\nabla} + e\vec{A}) \cdot \vec{A}_{e-} = 0. \quad (4b)$$

On setting $\Phi_{e+} = \psi$, $\vec{A}_{e+} = \vec{\sigma}\chi$, $\Phi_{e-} = \chi$, $\vec{A}_{e-} = \vec{\sigma}\psi$ we obtain Dirac's equation

Identically if the carrier-wave energy is equal to the rest-mass energy $\hbar\omega_e = m_e c^2$,

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi - \hbar\omega_e\right)\psi + \vec{\sigma} \cdot (i\hbar c\vec{\nabla} + e\vec{A})\chi = 0 \quad (5a)$$

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi + \hbar\omega_e\right)\chi + \vec{\sigma} \cdot (i\hbar c\vec{\nabla} + e\vec{A})\psi = 0, \quad (5b)$$

where $\vec{\sigma}$ is Pauli's vector. Unlike the classical electromagnetic potentials, which are real, the electron's potentials are complex. This is obvious when we notice that the + and - envelopes are not complex or Hermitian conjugates of one another.

An electromagnetic contribution to the mass of the electron due to the quantum radiation field associated with its motion is a well known concept in QED. Indeed the carrier-wave frequency of the electron's four-potential [Eqs. (3)] is equal to mc^2/\hbar , which is the high-frequency cut off for the quantum radiation field assumed in QED atomic structure calculations. The present derivation of Dirac's equation suggests that the total mass of the electron is electromagnetic in nature. This result is consistent with a previous result in which the charge of the electron was derived from Maxwell's equations [7].

3. Adiabatic nature of Dirac's solution of his equation

Although the time-dependent Dirac equation can be written in the Schroedinger form,

$$i\hbar \frac{\partial \psi_D}{\partial t} = H_D \psi_D, \quad (6)$$

where H_D is the Dirac Hamiltonian and $\psi_D(\vec{r}, t)$ is Dirac's four-component vector wave function, it does not follow that the energy-domain equation can be written in the Schroedinger form,

$$E\psi_{D,E} = H_D \psi_{D,E}, \quad (7)$$

unless one requires that all components of $\psi_D(\vec{r}, t)$ oscillate in time at a single frequency

$\omega = \frac{E}{\hbar}$, such that $\psi_D(\vec{r}, t)$ has the harmonic form $\psi_D(\vec{r}, t) = e^{-i\frac{E}{\hbar}t} \psi_{D,E}(\vec{r})$. The requirement does not hold in the general time-dependent solution of a vector wave function whose components are temporally coupled.

Eqs. (5) are rewritten in the standard Dirac form,

$$(i\hbar \frac{\partial}{\partial t} - e\Phi - mc^2)\psi + \vec{\sigma} \cdot (i\hbar c \vec{\nabla} + e\vec{A})\chi = 0 \quad (8a)$$

$$(i\hbar \frac{\partial}{\partial t} - e\Phi + mc^2)\chi + \vec{\sigma} \cdot (i\hbar c \vec{\nabla} + e\vec{A})\psi = 0, \quad (8b)$$

where $\vec{\sigma}$ is Pauli's vector and ψ, χ are the large, small components of Dirac's four-component wave function ψ_D . Eq. (8b) can be eliminated exactly in favor of Eq. (8a) as follows,

$$(i\hbar \frac{\partial}{\partial t} - e\Phi - mc^2)\psi(\vec{r}, t) = i\hbar c^2 \vec{\sigma} \cdot \vec{\nabla} \int_0^t dt' e^{\frac{i}{\hbar}(mc^2 - e\Phi)(t-t')} \vec{\sigma} \cdot \vec{\nabla} \psi(\vec{r}, t'), \quad (9)$$

where we have specialized to an electromagnetic field free problem by setting $\vec{A} = 0$.

Dirac's energy-domain solution is obtained by substituting $\psi(\vec{r}, t) = e^{-i\frac{E}{\hbar}t} \psi_E(\vec{r}, t)$ and assuming that $\psi_E(\vec{r}, t)$ is slowly varying in the time compared to the exponential factor such that the integral is evaluated approximately by holding $\psi_E(\vec{r}, t')$ constant at $t' = t$. Then the integration is performed, and the rapidly oscillating lower-limit contribution is dropped as small compared to the stationary upper-limit contribution. Such approximations to solve coupled time-dependent equations are known in the optical-physics literature as adiabatic elimination. Dirac's second-order equation for the large component follows immediately,

$$[(E - e\Phi)^2 - (mc^2)^2]\psi_E(\vec{r}) = -(\hbar c)^2 [\nabla^2 + \frac{1}{E - e\Phi + mc^2} [(\vec{\nabla} e\Phi) \cdot \vec{\nabla} + i\vec{\sigma} \cdot (\vec{\nabla} e\Phi \times \vec{\nabla})]]\psi_E(\vec{r}) \quad (10)$$

where we have used the identity, $(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$ and the time has been dropped from the argument list since the approximations to the t' integral render the wave

function stationary. Clearly Dirac's use of the Schroedinger forms $\psi(\vec{r}, t) = e^{-i\frac{E}{\hbar}t} \psi_E(\vec{r})$ and

$\chi(\vec{r}, t) = e^{-i\frac{E}{\hbar}t} \chi_E(\vec{r})$ to write the energy-domain form of his coupled time-dependent equations [Eqs. (8)] rests on an implicit assumption that adiabatic elimination of one of these equations in favor of the other is an accurate approximation. On other words the Schroedinger form does not hold exactly in the case of a vector wave function whose components are temporally coupled.

Dirac's harmonic ansatz for his time-dependent equation gives him a energy-domain equation which is exactly solvable for the free-electron and Coulomb problems. The Schroedinger form of the temporal solution, which is exact for Schroedinger's scalar wave equation but not for Dirac's vector wave equation, is in effect a form of calibration of Dirac theory to Schroedinger theory and has cast Dirac theory in the limited role of "correcting" Schroedinger theory primarily for relativistic effects in atomic structure. Probably as a result of its restricted use in electron physics, time-domain Dirac theory until recently had not been used to discover the a priori physical basis for Fermi-Dirac statistics [8], which is a spin-dependent phenomenon. The history of quantum mechanics instead followed a path of ensuring that Schroedinger wave functions satisfy Fermi-Dirac statistics on the basis of experimental observation and not a priori theory by using the Slater determinantal wave function to solve Schroedinger's wave equation for many electrons, even though Schroedinger theory, in which particle spin is absent, contains no physical basis for Fermi-Dirac statistics. One must instead turn to time-domain Dirac theory and the Dirac current to discover the physical basis for Fermi-Dirac statistics, which is elucidated using spin-dependent quantum trajectories [8]. Richard Feynman [9] once asked if spin is a relativistic requirement and then answered in the negative because the Klein-Gordon equation is a valid relativistic equation for a spin-0 particle. The correct answer is that spin is a relativistic requirement to insure Lorentz invariance in a vector-wave theory such as the Dirac or Maxwell theories. In the sense that Fermi-Dirac statistics depends critically on spin and yet is a phenomenon of order $(Zc)^0$, where c is the speed of light and Z is the atomic number, it would appear that authors [10] are misguided who present the quantum theory of matter as fundamentally based on Schroedinger theory as augmented by Dirac theory for "relativistic corrections" of order Z^4c^{-2} due to the acceleration of an electron moving near a nucleus with atomic number Z .

4. General solution of Dirac's time-domain wave equation

In this section the general time-dependent solution is presented free of any harmonic bias.

Solving the Coulomb problem ($e\Phi = V = -\frac{Ze^2}{r}$) the radial form of Eq. (9),

$$(i\hbar \frac{\partial}{\partial t} - V - mc^2)G_{\kappa}(r, t) = i\hbar c^2 \left(\frac{\partial}{\partial r} - \frac{(\kappa-1)}{r} \right) \int_0^t dt' e^{\frac{i}{\hbar}(mc^2 - V)(t-t')} \left(\frac{\partial}{\partial r} + \frac{(\kappa+1)}{r} \right) G_{\kappa}(r, t') \quad (11)$$

follows from the well-known substitutions,

$$\psi(\vec{r}, t) = G_{\kappa}(r, t) \chi_{\kappa l}(\theta, \phi) \quad (12a)$$

$$\vec{\sigma} \cdot \vec{\nabla} = \vec{\sigma} \cdot \hat{r} \left(\frac{\partial}{\partial r} - \frac{1}{r} \vec{\sigma} \cdot \vec{L} \right) \quad (12b)$$

$$\vec{\sigma} \cdot \hat{r} \chi_{\kappa l} = -\chi_{-\kappa l} \quad (12c)$$

$$\vec{\sigma} \cdot \vec{L} \chi_{\kappa l} = -(\kappa + 1) \chi_{\kappa l} \quad (12d)$$

where the angular functions are Dirac's two-component spinors. Eq. (11) is solved numerically in the variables r and ct for the hydrogen-like ground state ($\kappa = -1$) with $Z=70$, starting for mathematical convenience with a Schroedinger wave function at initial time and using the trapezoid rule to evaluate the integral. It is found that the evolved wave function is insensitive to the starting function at initial time.

At the point $t=t'$ the Crank-Nicolson implicit integration procedure is used in order to insure that the time integration of the equation itself is unconditionally stable. Fig. 1 shows the spectrum of states calculated from the inverse temporal Fourier transform of the wave function [11-12]. The spectrum has a strong peak in the positive-energy regime and a weak peak in the negative-energy regime, which lies in the negative-energy continuum and thus accounts for the unbound tail (Fig. 2). This temporally expanding tail appears to be the Coulomb counterpart of the Zitterbewegung solution calculated by Schroedinger [13] using the time-dependent Dirac equation for a free electron.

Fig. 2 shows the real part of radial wave function times r . Notice that the wave function is unusual in that it behaves like a bound state close to the nucleus but yet is unbound with a small-amplitude tail along the r axis whose length is equal to ct . In other words the tail propagates away from the nucleus at the speed of light. Nevertheless I have normalized the wave function for unit probability of finding the electron within a sphere of radius r_{\max} . The amplitude of the interior portion flows with time between the real part (Fig. 2) and imaginary part of the wave function such that the probability density is steady within the radius of the atom. $(ct)_{\max}$ is chosen to be three-fourths of r_{\max} in order that the propagating piece of the wave function stays well away from the grid boundary at r_{\max} . Calculations show that the results are insensitive to r_{\max} and therefore to $(ct)_{\max}$ as long as r_{\max} is well outside the region represented by the bound piece of the wave function, that is well outside of the radius of the atom as represented by standard Dirac theory. Notice that if the dynamical calculation were extended to very large times, then the wave function would fill a very large volume. In principle after a sufficient time the wave function could fill a volume the size of the universe although its interior part would remain the size of an atom.

What is the physical interpretation of Zitterbewegung? In view of the Maxwell-Dirac equivalency elucidated in Section II, we postulate here that it is a photonic energy of order $2mc^2$, which is the energy gap between the positive- and negative-energy electron continua and which was identified in Section II as an electromagnetic carrier-wave energy equal to $2\hbar\omega$. This amount of energy must be carried away from the atom in a continuous sense since there is no net loss of interior probability density over time. The energy originates from the electron's simultaneous double occupancy of both positive- and negative-energy

states (Fig. 4) whose energy difference is of order $2mc^2$. In standard Dirac theory the positive- and negative-energy levels are dynamically uncoupled such that Dirac assumed that electrons exclusively occupy the positive-energy levels and that the atom was stabilized by a set of negative-energy levels – the negative-energy sea – which are totally filled with electrons such that the Pauli Exclusion Principle forbids the downward fall of an electron from positive- to negative-energy levels accompanied by the emission of a photon with energy of order $2mc^2$.

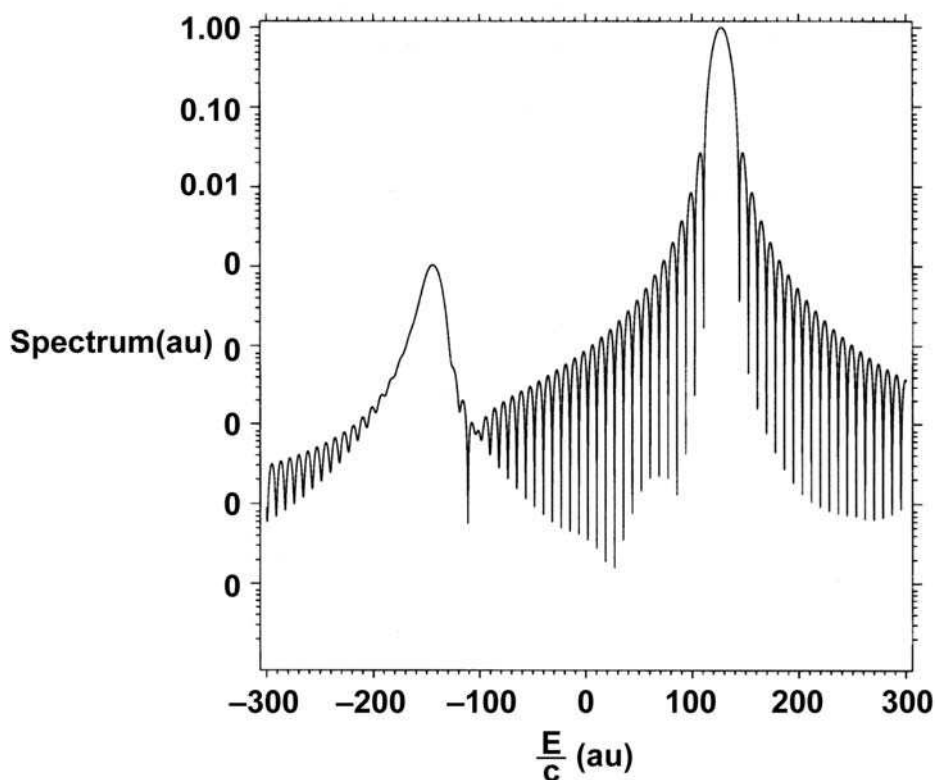


Fig. 1. Spectrum showing weak coupling of the positive- and negative-energy regions. The continuum edges are at $E/c = \pm mc$ au. The energy is obtained by multiplying the graphical numbers by c . A blow up of the positive energy peak shows good agreement with the eigenvalue at 17474.349, although the spectral calculation, because of the nature of the spectral determination of the eigenenergy, is not good to the number of significant figures shown.

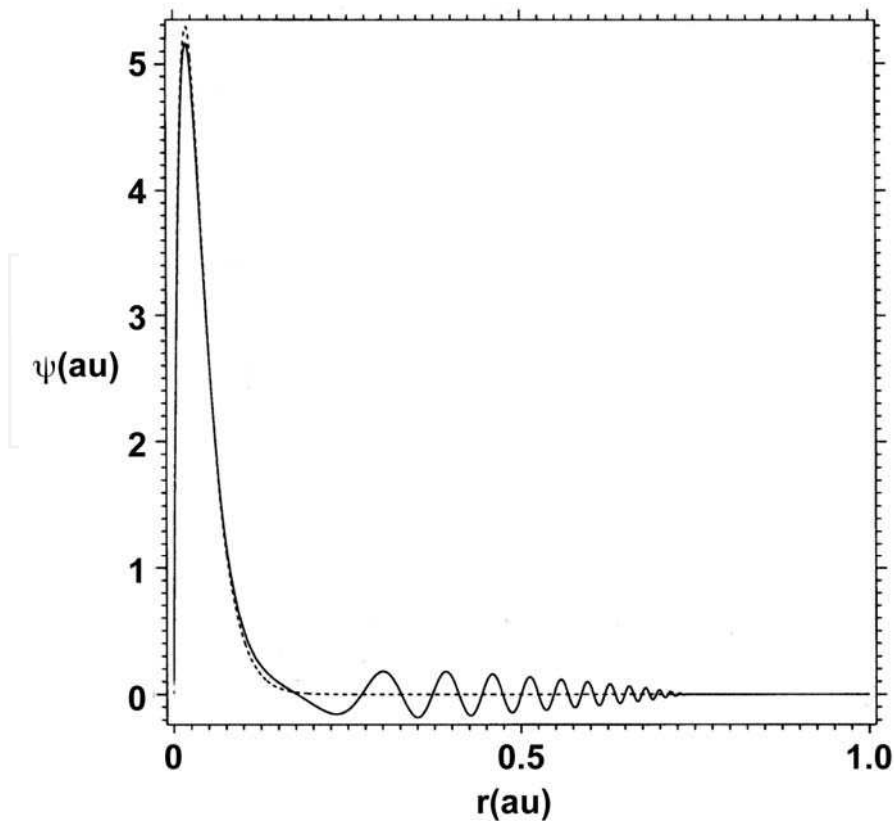


Fig. 2. Solid: imaginary part of the solution of Eq. (6) times r for $Z=50$, $(ct)_{\max}=0.75r_{\max}=0.75$ au. The number of ct , r grid points is 20K, 20K. Dotted: radial solution of Eq. (5) times r . The eigenvalue is found from the zero wronskian of forward and backward integrations and is equal to 17474.349 au to the number of significant figures shown in agreement with the

analytic Dirac energy $\varepsilon = \frac{mc^2}{\sqrt{1 + \left[\frac{\alpha Z}{\sqrt{1 - (\alpha Z)^2}} \right]^2}}$ where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant.

Dotted: wave function calculated from the radial equation inferred from Eq. (5).

In the general time-domain solution presented here it appears that the atom is self-stabilizing due to the mixed material-electromagnetic nature of the electron. Recall that in Section II we postulated that the electron's equation of motion should be the scalar product of its material four-momentum and its electromagnetic four potential. The solution of the equation of motion shows that the electron can share two ground-state material energy levels with energy conservation and without temporal decay of its quantum state as long as the energy difference between the two ground-state levels is converted to the energy of a continuously-emitted photon.

5. Photon equations of motion

In this section equations of motion for the photon are given and used to calculate a divergence-free Lamb shift [14-15]. As in the case of the electron in Section II we assume that a complex four-potential exists for the photon such that a photon EOM can be written as the Lorentz invariant formed by taking the scalar product of the photon's four-momentum and the photon's four-potential,

$$\left(\frac{\hbar}{c} \frac{\partial}{\partial t}, \hbar \vec{\nabla} - \frac{e\hbar}{mc^2} \vec{E}, \vec{H}\right) \cdot (\Phi_{\nu}, \vec{A}_{\nu}) = \frac{\hbar}{c} \frac{\partial}{\partial t} \Phi_e + \left(\hbar \vec{\nabla} - \frac{e\hbar}{mc^2} \vec{E}, \vec{H}\right) \cdot \vec{A}_{\nu} = 0, \quad (13)$$

for either electric or magnetic fields \vec{E}, \vec{H} . The photon four-momentum was found in [14] from \hbar times a form of the four-gradient whose scalar product with the four-electromagnetic-energy density gives the electromagnetic continuity equation. This is simply the electromagnetic analog of writing the material continuity equation as the scalar product of the four-gradient and the material four-density.

The electron scalar and vector potentials can be written in the form of carrier-wave expansions,

$$\Phi_{\nu} = \Phi_{\nu-} e^{-i\omega_{\nu} t} + \Phi_{\nu+} e^{i\omega_{\nu} t} \quad (14a)$$

$$\vec{A}_{\nu} = \vec{A}_{\nu-} e^{-i\omega_{\nu} t} + \vec{A}_{\nu+} e^{i\omega_{\nu} t}, \quad (14b)$$

from which on substituting Eqs. (14) into Eq. (13) and separately setting the coefficients of the exponential factors equal to zero, we obtain,

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + i \frac{\omega_{\nu}}{c}\right) \Phi_{\nu+} + \left(\vec{\nabla} - \frac{e}{mc^2} \vec{E}, \vec{H}\right) \cdot \vec{A}_{\nu+} = 0 \quad (15a)$$

$$\left(\frac{1}{c} \frac{\partial}{\partial t} - i \frac{\omega_{\nu}}{c}\right) \Phi_{\nu-} + \left(\vec{\nabla} - \frac{e}{mc^2} \vec{E}, \vec{H}\right) \cdot \vec{A}_{\nu-} = 0. \quad (15b)$$

On setting $\Phi_{\nu+} = \xi_{E,H}$, $\vec{A}_{\nu+} = \vec{\sigma} \xi_{E,H}$, $\Phi_{\nu-} = \zeta_{E,H}$, $\vec{A}_{\nu-} = \vec{\sigma} \zeta_{E,H}$ we obtain the Dirac form for the photon EOM presented previously assuming zero photon mass ($\hbar\omega_{\nu} = 0$),

$$\frac{\partial \xi_{E,H}}{c \partial t} + i \frac{\omega_{\nu}}{c} \xi_{E,H} + \vec{\sigma} \cdot \left(\vec{\nabla} - \frac{e}{mc^2} \vec{E}, \vec{H}\right) \zeta_{E,H} = 0 \quad (16a)$$

$$\frac{\partial \zeta_{E,H}}{c \partial t} - i \frac{\omega_{\nu}}{c} \zeta_{E,H} + \vec{\sigma} \cdot \left(\vec{\nabla} - \frac{e}{mc^2} \vec{E}, \vec{H}\right) \xi_{E,H} = 0 \quad (16b)$$

Writing $\xi_{E,H} = e^{-i\omega t} \psi_{E,H}$ and $\zeta_{E,H} = e^{-i\omega t} \chi_{E,H}$ in Eqs. (16) we derive stationary equations for $\psi_{E,H}$ and $\chi_{E,H}$; then we eliminate the equation for $\chi_{E,H}$ in favor of a second-order equation for $\psi_{E,H}$, obtaining equations for the electric and magnetic photon wave functions which have the Helmholtz form,

$$\left\{ \nabla^2 + \frac{\omega^2 - \omega_{\nu}^2}{c^2} - \frac{e}{mc^2} [\vec{\nabla} \cdot \vec{E} + 2\vec{E} \cdot \vec{\nabla} + i\vec{\sigma} \cdot (\vec{\nabla} \times \vec{E})] - \frac{e}{mc^2} E^2 \right\} \psi_E = 0 \quad (17a)$$

$$\left\{ \nabla^2 + \frac{\omega^2 - \omega_v^2}{c^2} - \frac{e}{mc^2} [\vec{\nabla} \cdot \vec{H} + 2\vec{H} \cdot \vec{\nabla} + i\vec{\sigma} \cdot (\vec{\nabla} \times \vec{H}) - \frac{e}{mc^2} H^2] \right\} \psi_H = 0, \quad (17b)$$

where we have used the identity, $(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$. Eq. (17b) for $\hbar\omega_v = 0$ was used in previous work to calculate the Lamb shift [14] and anomalous magnetic moment [16].

6. Subatomic bound states

Dirac's time-domain equation can be cast in the form of an equation second order in space and time; thus we should expect a second spatial-temporal solution to exist which is independent of the first spatial-temporal solution which we have elucidated in Section IV. I show that a regime exists in which an adiabatic solution to the time-dependent Dirac equation is not justified even in an approximate sense. The existence of the regime is easily recognized by writing Dirac equations in the form given by Eq. (11) for the large component with a reversal of charge and for the small component with no reversal of charge and then seeking solutions for which the phase in the exponential factor vanishes for all times. These equations are,

$$(i\hbar \frac{\partial}{\partial t} \mp |V| \mp mc^2) H_{\kappa}^{\pm}(r, t) = i\hbar c^2 \left(\frac{\partial}{\partial r} \mp \frac{(\kappa \mp 1)}{r} \right) \int_0^t dt' e^{\pm \frac{i}{\hbar}(mc^2 - |V|)(t-t')} \left(\frac{\partial}{\partial r} \pm \frac{(\kappa \pm 1)}{r} \right) H_{\kappa}^{\pm}(r, t'). \quad (21)$$

Eqs. (21) are solved numerically for $Z=1$ and $\kappa = \pm 1$ using the same techniques used to solve Eq. (11). The two equations for positronic or electronic binding are solved for a wave function or its complex conjugate respectively. The spectrum is found to be given simply by $E_{\pm} = \pm mc^2$ (Fig. 3). The real part of the wave function is shown in Fig. 4. Notice that if a bound state exists for one charge, then a bound state must also exist for the other charge by the charge-conjugation symmetry of Dirac's equation. Charge-conjugation symmetry is well known in standard time-independent Dirac theory, whose adiabatic regime does not support positronic-electronic bound states, and arises in Dirac's interpretation of the negative-energy states in which a hole or absence of an electron registers the existence of a positron or conversely in a positron world the absence of a positron would signal the existence of an electron.

Although the wave function is pulled inward toward the origin, its extent is still large compared to the radius of the proton $r_p = 1.3 \times 10^{-13} \text{ cm} = 2.46 \times 10^{-5} \text{ au}$.

The spectral energies are those which cancel the terms $\mp mc^2$ on the left side of Eqs. (21) and for which the stationary phases on the right side occur at $2mc^2 - |V| = 0$. For the unit-strength Coulomb potential the radius at which the stationary-phases occur is given by

$$r_{sp} = \frac{e^2}{2mc^2}, \text{ which is roughly the radius of the proton.}$$

The bound behavior of the positronic-electronic wave function shown in Fig. 4 can be understood as follows. Recognizing that the first and third terms on the left side of Eq. (21) cancel from the spectral values $E_{\pm} = \pm mc^2$ (Fig. 3), one may write an equation in which zero phase of the integration factor is assumed and which is the time derivative of both sides of Eq. (21) with zero phase,

$$\mp |V| \frac{\partial f}{\partial t} = i\hbar c^2 \left(\frac{\partial^2 f}{\partial r^2} + \frac{2}{r} \frac{\partial f}{\partial r} - \frac{2}{r^2} f \right), \quad (22)$$

A solution to Eq. (22) is sought in the form $f(r, t) = e^{i\omega t} g(r)$ for the complex separation constant $\omega_r + i\omega_i$, giving the equation for g ,

$$\frac{\partial^2 g}{\partial r^2} + \frac{2}{r} \frac{\partial g}{\partial r} - \frac{2}{r^2} g \pm \frac{|V|(\omega_r + i\omega_i)}{\hbar c^2} g = 0. \quad (23)$$

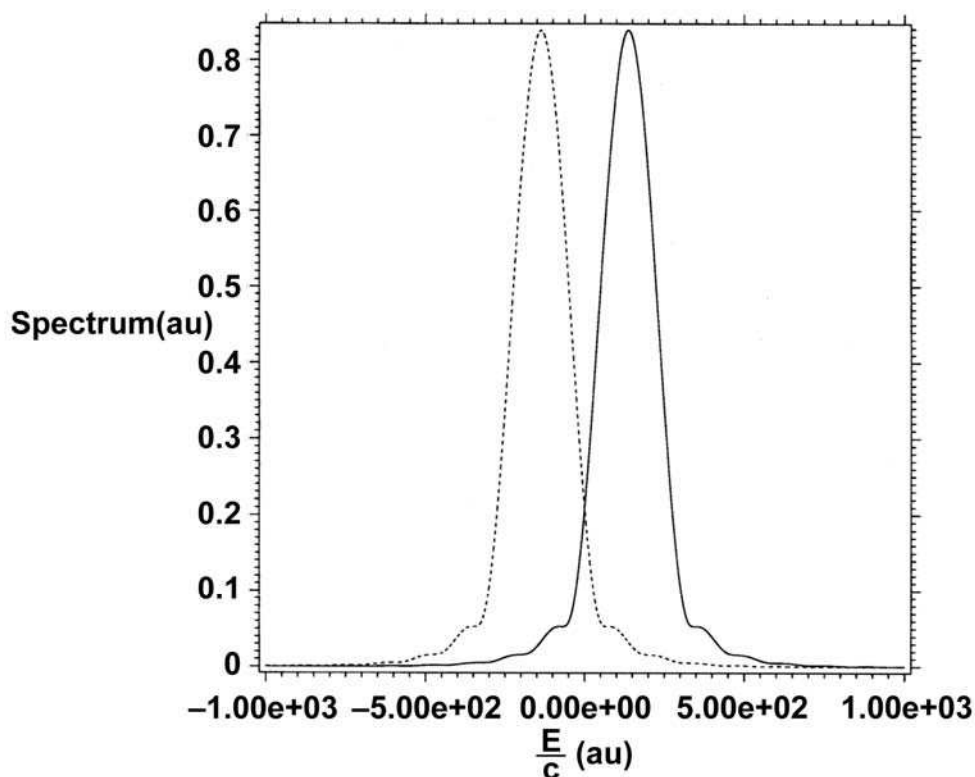


Fig. 3. Spectra from the solution of Eq. (21) using $r_{\max}=0.1$ au. Solid: positive charge. Dashed: negative charge. The continuum edges are at $E/c = \pm mc$ au. The energy is obtained by multiplying the graphical numbers by c .

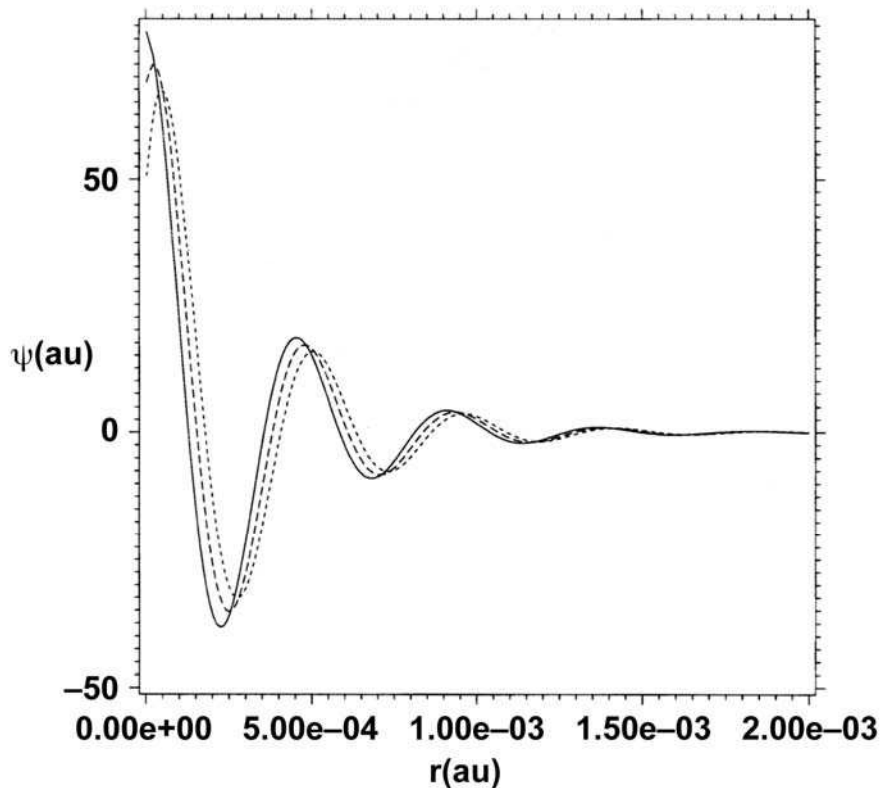


Fig. 4. Real part of the positronic or electronic solution of Eq. (21) times r at $ct=0.0375$ (solid), 0.0750 (dashed), and 0.1125 (dotted) au showing the convergence to a stationary solution. The initial wave function, which is hydrogenic and spread out in the domain $0.25 \times 10^{-5} < r < 0.2$ au, is pulled into the origin as shown in the figure.

Figs. 5-6 show plots of the real part of f and of the real and imaginary parts of g respectively for $r=ct$ and $\omega_r = -92 \frac{mc^2}{\hbar}$, and $\omega_i = 35 \frac{mc^2}{\hbar}$. Except for the behavior near the origin the unnormalized solution of Eq. (23) is a good mimic of the solution of Eq. (21) shown in Fig. 3. Remarkably the bound positronic-electronic states in the nonadiabatic regime (Fig. 4) exhibit an altogether different form of binding than that of Schrödinger or time-independent Dirac theory. This is obvious from the spectrum (Fig. 3), in which the energies lie at the edges of the positive- and negative-energy continua. One may understand this form of binding as binding which satisfies the four-space Lorentz-invariant relationship $r^2 - (ct)^2 = 0$ between position and time. In other words the binding can occur as a temporal exponential decay in which $ct = r$ rather than as a spatial exponential decay requiring eigenvalues which fall somewhere in the gap between the two continua. This point is clear from Figs. 5-6 in which binding occurs in the temporal part of the function $f(r,t)$ (Fig. 5) while the radial function $g(r)$ is unbound (Fig. 6).

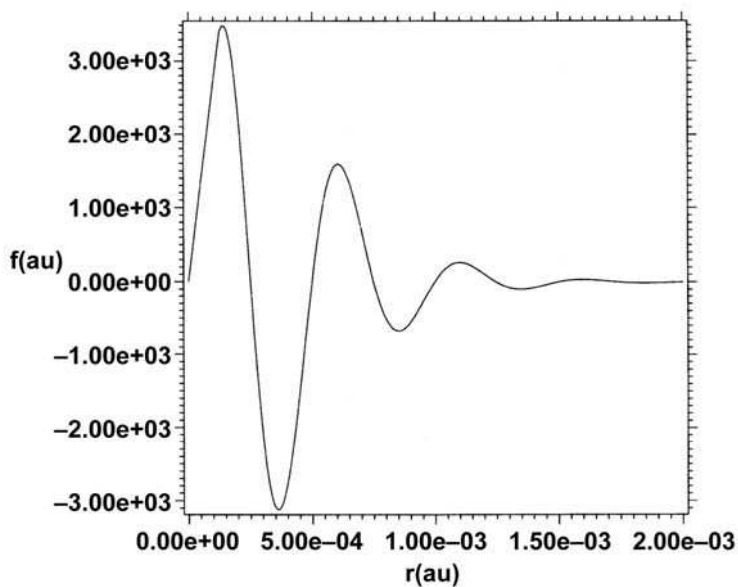


Fig. 5. Simulation using Eqs. (22)-(23) of the wave function shown in Fig. 4. The simulated wave function is unnormalized.

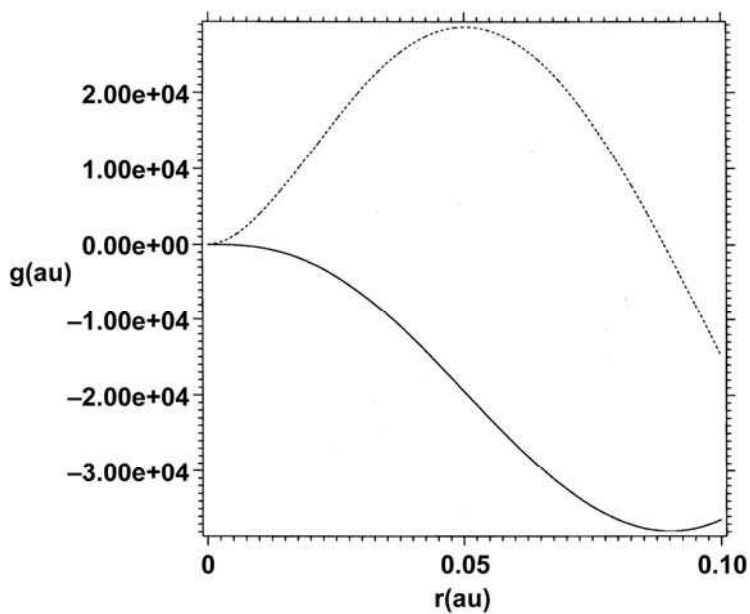


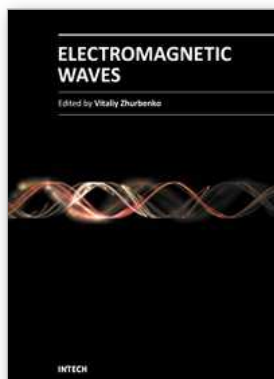
Fig. 6. Unnormalized wave function obtained from Eq. (23) by outward integration. Solid: real part. Dashed: imaginary part.

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