10

Models for Scattering from Rough Surfaces

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1. Introduction

Models for scattering of electromagnetic waves from random rough surfaces have been developed during the last two centuries and the scientific interest in the problem remains strong also today due to the importance of this phenomenon in diverse areas of science, such as measurements in optics, geophysics, communications and remote sensing of the Earth.
Such models can be categorised into empirical models, analytical models and a combination of the two. Though very simple, empirical models are greatly dependent on the experimental conditions. In spite of their complexity, only theoretical models can yield a significant understanding of the interaction between the electromagnetic waves and the Earth's surface, although an exact solution of equations governing this interaction may not always be available and approximate methods have to be used. The semi-empirical models, which are based on both physical considerations and experimental observations, can be set between these two kinds of models and can be easily inverted. In this survey, we will focus on the analytical models and we study more in detail the *Kirchhoff Approximation* (KA), the *Small Perturbation Method* (SPM) and the *Integral Equation Method* (IEM). The Kirchhoff Approximation and the Small Perturbation Methods represent early approaches to scattering which are still much used, whereas the Integral Equation Method represents a newer approach which has a larger domain of validity. These methods have been found to be the most common in the literature and many of the other methods are based or have much in common with these approaches. In section 2, we begin by giving a brief presentation of the scattering problem and introduce some concepts and results from the theory of electromagnetic fields which are often used in this context. We will also define the bistatic scattering coefficient, due to the importance of this type of measurement in many remote sensing applications, and in particular in the retrieval of soil moisture content. In section 3, we give a brief presentation on the Kirchhoff Approximation and its close variants, the *Physical Optics* (PO) and the *Geometrical Optics* (GO). In section 4, we give a brief presentation of the Small Perturbation Method and in section 5 we will present the Integral Equation Model.

2. Some concepts of the electromagnetic theory and surface parameters

In this section we will give a brief presentation of some concepts on theories of electromagnetism and statistical characterisation of surfaces, which are often used for

modelling scattering of electromagnetic waves from random rough surfaces. We will also define the bistatic scattering coefficient due to the importance of this type of measurement in many remote sensing applications.

2.1 The Maxwell's equations and the wave equation

The basic laws of the electromagnetism are given by the Maxwell's equations which, for linear, homogeneous, isotropic, stationary and not dispersive media, can be written as (Balanis, 1989):

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.1.1}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_c + \mathbf{J}_i \tag{2.1.2}$$

$$\nabla \cdot \mathbf{D} = \rho \tag{2.1.3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.1.4}$$

where **E** is the electric field vector, **D** is the electric flux density, **H** is the magnetic field vector, **B** is the magnetic flux density, **J** is the conduction electric current density, $\mathbf{J}_i$ is the impressed electric current density and ρ is the electric charge density. Maxwell's equations together with the boundary conditions, give a complete description of the field vectors at any points (including discontinuities) and at any time. In rough surface scattering, the surface enters in the boundary conditions (see equations (2.2.1)-(2.2.4)), which have to be also supplied at infinity.

If we consider *time-harmonic* variation of the electromagnetic field, the instantaneous field vectors can be related to their complex forms. Thus the Maxwell's equations can be written in a much simpler form:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{2.1.5}$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} + \mathbf{J}_i \equiv j\omega\varepsilon_c\mathbf{E} + \mathbf{J}_i \tag{2.1.6}$$

$$\nabla \cdot \varepsilon\mathbf{E} = \rho \tag{2.1.7}$$

$$\nabla \cdot \mu\mathbf{H} = 0 \tag{2.1.8}$$

where we assumed the region characterised by permeability μ, permittivity ε and conductivity σ (lossy medium). To obtain the governing equation for the electric field, we take the curl of (2.1.5) and then replace (2.1.6). Thus,

$$\nabla \times \nabla \times \mathbf{E} + \omega^2\mu\varepsilon_c\mathbf{E} = j\omega\mu\mathbf{J}_i \tag{2.1.9}$$

which is known as the inhomogeneous Helmholtz vector wave equation. In a free-source region, $\nabla \cdot \mathbf{E} = 0$ and (2.1.9) simplifies to:

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon_c\mathbf{E} = 0 \tag{2.1.10}$$

In rectangular coordinates, a simple solution to (2.1.10) has the form:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \tag{2.1.11}$$

where $\mathbf{E}_0$ is a constant complex vector which determines the polarisation characteristics and the complex propagation vector, $\mathbf{k}$, is defined as:

$$\mathbf{k} = \hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z \tag{2.1.12}$$

with the components satisfying

$$k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon_c = k^2 \tag{2.1.13}$$

Equation (2.1.11) represents a plane wave and k is the propagation constant. Most analytical methods for scattering from rough surfaces assume this kind of incident wave, which if linearly polarised can be rewritten as:

$$\mathbf{E}^i(\mathbf{r}) = \hat{\mathbf{p}}E_0 e^{-j\mathbf{k}_i\cdot\mathbf{r}} = \hat{\mathbf{p}}E^i \tag{2.1.14}$$

where $\mathbf{k}_i = \hat{\mathbf{k}}_i k$, $\hat{\mathbf{p}}$ is the unit polarisation vector and E_0 is the amplitude. The associated magnetic field is given by:

$$\mathbf{H}^i(\mathbf{r}) = \hat{\mathbf{k}}_i \times \mathbf{E}^i(\mathbf{r})/\eta \tag{2.1.15}$$

where $\eta = \sqrt{\mu/\varepsilon_c}$ is the wave impendence in the medium.

2.2 Integral theorems and other results used in scattering models

We will present some results for electromagnetic fields which are often used as a starting point in the analytical models for scattering from rough surfaces. These equations are approximated and simplified using different methods and assumptions in the analytical solutions for scattering from rough surfaces. We will not show how the equations in this section are derived, but derivation can be found in the references.

Consider an electromagnetic plane wave incident on a rough surface as shown in figure 2.2.1.

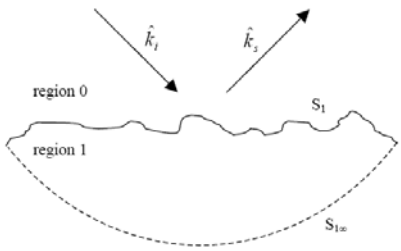

Fig. 2.2.1. Scattering of electromagnetic field on surface separating two media.

Across any surface interface, the electromagnetic field should satisfy continuity conditions given by (Balanis, 1989):

$$\hat{\mathbf{n}}\times\left(\mathbf{E}-\mathbf{E}_1\right)=0 \tag{2.2.1}$$

$$\hat{\mathbf{n}}\times\left(\mathbf{H}-\mathbf{H}_1\right)=\mathbf{J}_s \tag{2.2.2}$$

$$\hat{\mathbf{n}}\cdot\left(\varepsilon\mathbf{E}-\varepsilon_1\mathbf{E}_1\right)=\rho_s \tag{2.2.3}$$

$$\hat{\mathbf{n}}\cdot\left(\varepsilon\mathbf{H}-\varepsilon_1\mathbf{H}_1\right)=0 \tag{2.2.4}$$

where $\hat{\mathbf{n}}$ is the unit normal vector of the rough surface (pointing in the region 0). The electric surface current density, $\mathbf{J}_s$, and the charge surface density, ρ_s, at the rough interface are zero unless the scattering surface (or one of the media) is a perfect conductor.
Using the fact that the fields satisfy the Helmholtz wave equation (2.1.9), it can be shown that in the region 0, the electromagnetic fields **E** and **H**, satisfy Huygens' principle and the radiation boundary condition at infinity and **E** is given by (Ulaby et al, 1982; Tsang et al, 2000):

$$\mathbf{E}(\mathbf{r})=\mathbf{E}^i(\mathbf{r})+\int_{S_1}\left\{-j\omega\mu\,\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')\cdot\hat{\mathbf{n}}'\times\mathbf{H}(\mathbf{r}')-\nabla\times\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')\cdot\hat{\mathbf{n}}'\times\mathbf{E}(\mathbf{r}')\right\}ds' \tag{2.2.5}$$

where $\overline{\overline{\mathbf{G}}}$ is the dyadic Green function (to the vector Helmholtz equation) which is represented by:

$$\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')=\left(\bar{\bar{\mathbf{I}}}+\frac{\nabla\nabla}{k^2}\right)g(\mathbf{r},\mathbf{r}') \tag{2.2.6}$$

Here $\bar{\bar{\mathbf{I}}}$ is the unit dyadic and $g(\mathbf{r},\mathbf{r}')$ is the Green function that satisfies the scalar wave equation. It assumes the following expression:

$$g(\mathbf{r},\mathbf{r}')=\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{2.2.7}$$

In (2.2.5) the first term on the right-hand side represents the field generated by a current source in an *unbounded* medium with permittivity ε and permeability μ and corresponds to the incident field. Hence, the electromagnetic field in the region 0 is expressed as the sum of two contributions: one is given by the incident field $\mathbf{E}^i(\mathbf{r})$; the other contribution is given by the surface integrals that involve the tangential components $\mathbf{E}_t$ and $\mathbf{H}_t$ of the fields at the boundary S_1 (note that $\hat{\mathbf{n}}'\times\mathbf{E}=\hat{\mathbf{n}}'\times\mathbf{E}_t$ and $\hat{\mathbf{n}}'\times\mathbf{H}=\hat{\mathbf{n}}'\times\mathbf{H}_t$) and represents the scattered field due to the presence of surface.
The equation (2.2.5) constitutes the mathematical basis of Huygens' principle in vector form. According to this principle, the electromagnetic field in a source-free region ($\mathbf{J}=0$) is uniquely determined once its tangential components are assigned on the boundary of the region. However, since in the region 0, the existence of the impressed current **J** has been

assumed, the total electric field can be expressed as the sum of two terms, the incident and scattering ones:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^s(\mathbf{r}) \tag{2.2.8}$$

Thus, the scattered field can be written as:

$$\mathbf{E}^s(\mathbf{r}) = \int_{S_1} \left\{ -j\omega\mu \overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}') \cdot \hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}') - \nabla \times \overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}') \cdot \hat{\mathbf{n}}' \times \mathbf{E}(\mathbf{r}') \right\} ds' \tag{2.2.9}$$

If the observation point is in the far field region, the Green function in (2.2.9) can be simplified and the scattering field can be written as (Ulaby et al,1982; Tsang et al, 2000):

$$\mathbf{E}^s(\mathbf{r}) = K\mathbf{r} \times \int_{S_1} \left[\left(\hat{\mathbf{n}}' \times \mathbf{E}(\mathbf{r}') \right) - \eta \hat{\mathbf{r}} \times \left(\hat{\mathbf{n}}' \times \mathbf{H}(\mathbf{r}') \right) \right] e^{j\mathbf{k}\cdot\mathbf{r}'} ds' \tag{2.2.10}$$

where $K = -jke^{-jkr}/4\pi r$ and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of observation. The tangential surface fields $\hat{\mathbf{n}} \times \mathbf{E}$ and $\hat{\mathbf{n}} \times \mathbf{H}$ can be also expressed as (Poggio & Miller, 1973):

$$\hat{\mathbf{n}} \times \mathbf{E} = 2\hat{\mathbf{n}} \times \mathbf{E}^i - \frac{2}{4\pi} \hat{n} \times \int \varepsilon \, ds' \tag{2.2.11}$$

$$\hat{\mathbf{n}} \times \mathbf{H} = 2\hat{\mathbf{n}} \times \mathbf{H}^i + \frac{2}{4\pi} \hat{\mathbf{n}} \times \int \mathcal{H} ds' \tag{2.2.12}$$

and

$$\hat{\mathbf{n}} \times \mathbf{E}_t = -\frac{2}{4\pi} \hat{\mathbf{n}}_t \times \int \varepsilon_t \, ds' \tag{2.2.13}$$

$$\hat{\mathbf{n}} \times \mathbf{H}_t = \frac{2}{4\pi} \hat{\mathbf{n}}_t \times \int \mathcal{H}_t \, ds' \tag{2.2.14}$$

where

$$\varepsilon = jk\eta (\hat{\mathbf{n}}' \times \mathbf{H}') G_1 - (\hat{\mathbf{n}}' \times \mathbf{E}') \times \nabla' G_1 - (\hat{\mathbf{n}}' \cdot \mathbf{E}') \nabla' G_1 \tag{2.2.15}$$

$$\mathcal{H} = \frac{jk}{\eta} (\hat{\mathbf{n}}' \times \mathbf{E}') G_1 - (\hat{\mathbf{n}}' \times \mathbf{H}') \times \nabla' G_1 - (\hat{\mathbf{n}}' \cdot \mathbf{H}') \nabla' G_1 \tag{2.2.16}$$

$$\varepsilon_t = -\left[jk_2\eta_2 (\hat{\mathbf{n}}' \times \mathbf{H}') G_2 - (\hat{\mathbf{n}}' \times \mathbf{E}') \times \nabla' G_2 - (\hat{\mathbf{n}}' \cdot \mathbf{E}') \nabla' G_2 (1/\varepsilon_r) \right] \tag{2.2.17}$$

$$\mathcal{H}_t = -\left[\frac{jk_2}{\eta_2} (\hat{\mathbf{n}}' \times \mathbf{E}') G_2 - (\hat{\mathbf{n}}' \times \mathbf{H}') \times \nabla' G_2 - (\hat{\mathbf{n}}' \cdot \mathbf{H}') \nabla' G_2 (1/\mu_r) \right] \tag{2.2.18}$$

and $\hat{\mathbf{n}}$, $\hat{\mathbf{n}}'$, $\hat{\mathbf{n}}_t$, $\hat{\mathbf{n}}_t'$ are the unit normal vectors to the surface and $\hat{\mathbf{n}}_t = -\hat{\mathbf{n}}$, $\hat{\mathbf{n}}_t' = -\hat{\mathbf{n}}'$, $\hat{\mathbf{n}} \times \mathbf{E}$ and $\hat{\mathbf{n}} \times \mathbf{H}$ are the total tangential fields on the rough surface in the medium above the

separating interface; G_1 and G_2 are the Green's functions in medium above and below the interface, respectively, and $\varepsilon_r = \varepsilon_2/\varepsilon_1$, $\mu_r = \mu_2/\mu_1$, $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$ and $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$.

2.3 The nature of surface scattering

When an electromagnetic wave impinges the surface boundary between two semi-infinitive media, the scattering process takes place only at the surface boundary if the two media can be assumed homogeneous. Under such supposition, the problem at issue is indicated as *surface scattering* problem. On the other hand, if the lower medium is inhomogeneous or is a mixture of materials of different dielectric properties, then a portion of the transmitted wave scattered backward by the inhomogeneities may cross the boundary surface into the upper medium. In this case scattering takes place within the volume of the lower medium and it is referred to as *volume scattering*. In most cases both the scattering processes are involved, although only one of them can be dominant. In the case of bare soil, which will be assumed to be a homogeneous body, surface scattering is the only process taken into consideration.

When the surface boundary separating the two semi-infinitive media is perfectly smooth the reflection is in the specular direction and is described by the Fresnel reflection laws. On the other hand, when the surface boundary becomes rough, the incident wave is partly reflected in the specular direction and partly scattered in all directions. Qualitatively, the relationship between surface roughness and surface scattering can be illustrated through the example shown in Figure 2.3.1. For the specular surface, the angular radiation pattern of the reflected wave is a delta function centred about the specular direction as shown in Figure 2.3.1 (a). For the slightly rough surface (Figure 2.3.1 (b)), the angular radiation pattern consists of two components: a reflected component and a scattered component. The reflected component is again in the specular direction, but the magnitude of its power is smaller than that for smooth surface. This specular component is often referred to as the *coherent* scattering component. The scattered component, also known as the *diffuse* or *incoherent* component, consists of power scattered in all directions, but its magnitude is smaller than that of the coherent component. As the surface becomes rougher, the coherent component becomes negligible.

Note that the specular component represents also the mean scattered field (in statistical sense), whereas the diffuse component has a stochastic behaviour, associated to the randomness of the surface roughness.

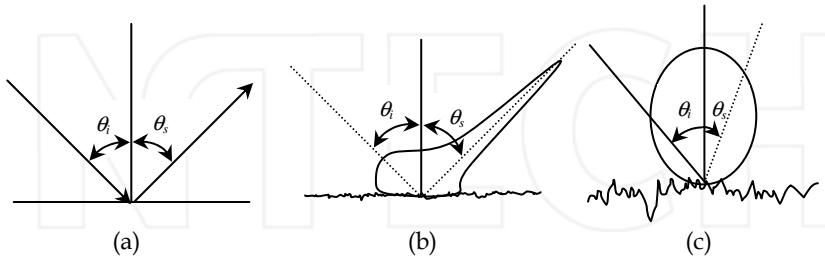

Fig. 2.3.1. Relative contributions of coherent and diffuse scattering components for different surface-roughness conditions: (a) specular, (b) slightly rough, (c) very rough.

2.3.1 Characterisation of soil roughness

A rough surface can be described by a height function $\varsigma = z(x, y)$. There are basically two categories of methods which are being used to measure surface roughness. The roughness

can be carried out by means of various experimental approaches able to reproduce the surface profile by using contact or laser probes, or it can be estimated using some theory which relates scattering measurements to surface roughness. In general, the study of scattering in remote sensing is performed by using random rough surface models, where the elevation of surface, with respect to some mean surface, is assumed to be an ergodic[1], and hence stationary[2], random process with a Gaussian height distribution.

Accordingly, the *degree of roughness*, or simply the *roughness*, of a random surface is characterised in terms of statistical parameters that are measured in units of wavelength. For this reason, a given surface that may "appear" very rough to an optical wave, may "appear" very smooth to a microwave.

The two fundamental parameters commonly used are the standard deviation of the surface height variation (or rms height) and the surface correlation length. Such parameters describe the statistical variation of the random component of surface height relative to a reference surface, that may be the unperturbed surface of a period pattern, as in the case of a row-tilled soil surface (Figure 2.3.1.1. (a)), or may be the mean plane surface if only random variations exist (Figure 2.3.1.1 (b)).

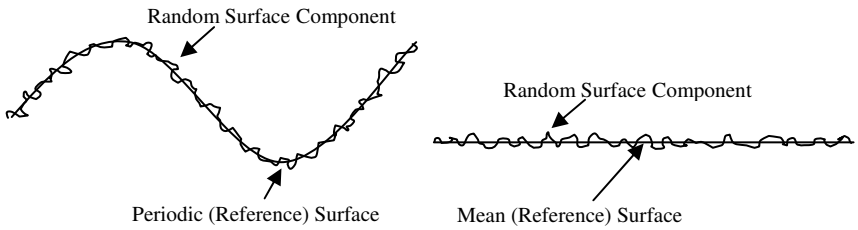

Fig. 2.3.1.1. Two configurations of height variations: (a) random height variations superimposed to a periodic surface; (b) random variations superimposed to a flat surface.

Let $z(x)$ be a representative realisation of the ergodic and stationary process that describes a generic rough surface in a one-dimensional case. The mean value, which throughout this chapter will be denoted by angular brackets $\langle ... \rangle$, is equal to the spatial average over a statistically representative segment of the surface, of dimensions L_x, centred at the origin:

$$\bar{z} = \frac{1}{L_x}\int_{-L_x/2}^{L_x/2} z(x)\,dx = \langle z(x) \rangle \tag{2.3.1.1}$$

As it can be noted from the above definition, for a stationary surface the average does not depend on x. The second moment is:

$$\overline{z^2} = \frac{1}{L_x}\int_{-L_x/2}^{L_x/2} z^2(x)\,dx = \langle z^2(x) \rangle \tag{2.3.1.2}$$

[1] A process is ergodic when one realisation is representative of all the process, i.e. the statistical averages over an extracted random variable may be replaced by spatial averages over a single realisation.

[2] The stationarity implies that all the statistically properties of a random process are invariant under the translation of spatial coordinates.

Using the above expressions, the standard deviation of the surface height, σ, is therefore defined as:

$$\sigma = \left[\overline{z^2} - \left(\overline{z}\right)^2 \right]^{1/2} = \left\langle \left[z(x) - \overline{z} \right]^2 \right\rangle^{1/2} \tag{2.3.1.3}$$

Such quantities characterise the dispersion of the surface height relative to the reference plane. Taking into account the stationary properties of the process and considering its mean value null, the variance, σ^2 , is coincident with the second moment and does not depend on x. The autocorrelation function of the height random process $z(x)$ is given by:

$$R_z(\tau) = \frac{1}{L_x} \int_{-L_x/2}^{L_x/2} z(x) z(x+\tau) dx = \left\langle z(x) z(x+\tau) \right\rangle \tag{2.3.1.4}$$

The normalised autocorrelation function (ACF), better known as the correlation coefficient, assumes for a process with zero mean value the following expression:

$$\rho(\tau) = \frac{\int_{-L_x/2}^{L_x/2} z(x) z(x+\tau) dx}{\int_{-L_x/2}^{L_x/2} z^2(x) dx} = \frac{R_z(\tau)}{R_z(0)} = \frac{R_z(\tau)}{\sigma^2} \tag{2.3.1.5}$$

It is a measure of the similarity between the height z at point x and at point distant τ from x. It has the following properties:

$$\left| \rho(\tau) \right| \le \rho(0) = 1 ;$$

$$\lim_{\tau \to \infty} \rho(\tau) = 0 .$$

The spectral density or power spectrum is defined, for an ergodic random process, as the Fourier transform of the autocorrelation function $R_z(x)$:

$$\tilde{W}(k_x) = \int_{-\infty}^{\infty} R_z(x) \, e^{jk_x x} dx \tag{2.3.1.6}$$

where k_x is the Fourier transform variable.
However, taking into account the equation (2.3.1.5), it is common practice in characterising the random surface to define the power spectrum of the normalised autocorrelation function:

$$W(k_x) = \int_{-\infty}^{\infty} \rho(x) \, e^{jk_x x} dx \tag{2.3.1.7}$$

The Gaussian distribution plays a central role in modelling scattering from random rough surfaces because it is encountered under a great number of different conditions and because Gaussian variates have the unique property that the random process is entirely determined by the height probability distribution and autocorrelation. All higher order correlations can be expressed in terms of the (second order) autocorrelation function, which simplifies modelling the surface scattering process. A simple and often used form for the

autocorrelation is the Gaussian function but other forms have also been studied (Saillard & Sentenac, 2001).

The roughness spectrum at the *n*'th power of the autocorrelation function, $W^{(n)}$, which often enters into closed form solutions of the scattering problem, is given by the Fourier transform:

$$W^{(n)}(k_x) = \int_{-\infty}^{\infty} \rho^n(x)\, e^{jk_x x} dx \tag{2.3.1.8}$$

The consideration of a realistic autocorrelation function is in fact a relevant problem for a better modelling of the soil scattering. Some often used forms (see for instance (Fung, 1994)) of the autocorrelation function are the Gaussian correlation function, the exponential correlation function, combinations of the Gaussian and exponential functions and the so called 1.5-power correlation function. For all of these, the roughness spectrum at the *n*'th power can be evaluated analytically (see (Fung, 1994)). For instance, for an isotropically rough surface, the normalised Gaussian autocorrelation in a single dimension assumes the following expression:

$$\rho(x) = \exp\left(-\frac{x^2}{l^2}\right) \tag{2.3.1.9}$$

where *l* is the *correlation length*. Such surface parameter is defined as the displacement *x* for which $\rho(x)$ is equal to $1/e$

$$\rho(l) = 1/e \tag{2.3.1.10}$$

The correlation length of a surface provides a reference for estimating the statistical independence of two points on the surface; if the two points are separated by a horizontal distance greater than *l*, then their heights may be considered to be (approximately) statistically independent of one another. In the extreme case of a perfectly smooth (specular) surface, every point on the surface is correlated with every other point with a correlation coefficient of unity. Hence, $l = \infty$ in this case.

Referring to equation (2.3.1.9), the *n*'th power roughness spectrum is equal to:

$$W^{(n)}(k_x) \equiv \int \rho^n(x) e^{jk_x x} dx = l\sqrt{\frac{\pi}{n}} e^{-\frac{k_x^2 l^2}{4n}} \tag{2.3.1.11}$$

Beside the height random function $z(x)$, the slope function is another important characterisation of the rough surface. It is defined as:

$$Z_x = \lim_{\Delta x \to 0} \frac{z(x+\Delta x) - z(x)}{\Delta x} \tag{2.3.1.12}$$

Considering the stationary random process $z(x)$ as normally distributed with zero mean and variance σ^2, being Z_x the first derivative, its distribution is again normal with zero mean and variance related to the second derivative of the autocorrelation function of $z(x)$ at the origin (Beckman & Spizzichino, 1963):

$$\sigma_s^2 = \left\langle Z_x^2 \right\rangle = -\sigma^2 \rho''(0) \tag{2.3.1.13}$$

The rms slope is subsequently indicated as *m*:

$$m = \left(-\sigma^2 \rho''(0)\right)^{1/2} \tag{2.3.1.14}$$

When the normalised autocorrelation function is Gaussian (equation (2.3.1.9)), the rms slope is equal to:

$$m = \sqrt{2}\frac{\sigma}{l} \tag{2.3.1.15}$$

2.4 Bistatic scattering coefficient

A quantity often used in models and measurements of scattering in the microwave region is the *bistatic scattering coefficient* $\sigma^o_{q,p}(\theta_i, \phi_i, \theta_s, \phi_s)$. It describes the target's scattering properties at a given frequency, polarisation, incidence and observing directions, being independent on the specific measurement system used. It is possible to define $\sigma^o_{q,p}$ directly in terms of the incident and scattering field E^i_p and E^s_q as follows (Ulaby et al, 1982):

$$\sigma^o_{q,p}(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{4\pi R^2 \left|E^s_p\right|^2}{A_0 \left|E^i_q\right|^2} \tag{2.4.1}$$

where the ensemble average must be considered in case the scattered field is the fluctuating zero mean component (i.e., the diffuse or incoherent component mentioned before) generated by a natural target or random rough surface. Such equation shows $\sigma^o_{q,p}$ as the ratio of the total power scattered by an equivalent isotropic scatterer in direction (θ_s, ϕ_s) to the product of the incident power density in direction (θ_i, ϕ_i) and the illuminated area.

The *backscattering coefficient* $\sigma^o_{q,p}(\theta_i)$ is a special case of $\sigma^o_{q,p}(\theta_i, \phi_i, \theta_s, \phi_s)$; it is defined for $\theta_s = \theta_i$ and $\phi_s = \phi_i \pm \pi$ (Figure 2.4.1), which corresponds to the incident and scattered direction being the same except for a reversal in sense.

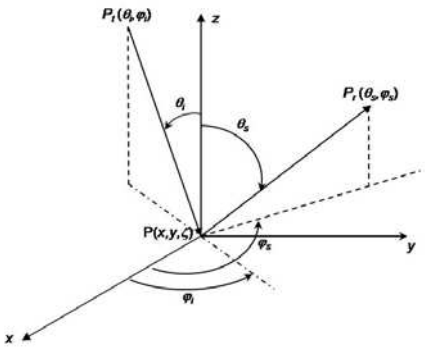

Fig. 2.4.1. Geometry of the scattering problem.

3. The Kirchhoff approximation

In this section we shall consider the Kirchhoff (also sometimes referred to as the tangent plane approximation) approach to describe the scattering from rough surfaces, which was one of the first methods applied. We will consider surfaces with random surface profiles (i.e. not period surfaces) and within the context of the vector theory we will discuss the Kirchhoff Approximation. We will consider here the case of scattering from 2-dimensional dielectric surfaces. We will present results for the case of a surface which can be characterised as a Gaussian random process. We will also mention some extensions of the Kirchhoff approximation and will give references to further reading about the Kirchhoff approach. The reference list is by no means complete, since the literature on the Kirchhoff approximation is vast. A good representation of the Kirchhoff method can be found for instance in (Tsang et al, 2000, Tsang & Kong, 2001, Ulaby et al, 1982).

3.1 Formulation of the scattering problem

The geometry of the scattering problem we consider is shown in figure 2.4.1. We consider a monochromatic, linearly polarised incident plane wave with electric and magnetic field given by the equations (2.1.14) and (2.1.15), respectively.

It can be shown, similarly to equation (2.2.10), that the far zone scattering field, E_{qp}^{s}, can be written in terms of the tangential surface fields in the medium above the separating surface as (Stratton-Chu integral) (Ulaby et al, 1982):

$$E_{qp}^{s} = K\int\left\{\hat{\mathbf{q}}\cdot\left[\hat{\mathbf{k}}_s\times\left(\hat{\mathbf{n}}\times\mathbf{E}_p\right)+\eta\left(\hat{\mathbf{n}}\times\mathbf{H}_p\right)\right]\right\}e^{j(\mathbf{k}_s\cdot\mathbf{r})}ds \tag{3.1.1}$$

where

$$\mathbf{k}_s = k\hat{\mathbf{k}}_s = k\left(\sin\theta_s\cos\phi_s\hat{\mathbf{x}}+\sin\theta_s\sin\phi_s\hat{\mathbf{y}}+\cos\theta_s\hat{\mathbf{z}}\right) = k_{sx}\hat{\mathbf{x}}+k_{sy}\hat{\mathbf{y}}+k_{sz}\hat{\mathbf{z}} \tag{3.1.2}$$

What needs to be calculated are the tangential surface fields in equation (3.1.1). In equations (2.2.11) - (2.2.12) and (2.2.15) - (2.2.16) we presented integral equations for the tangential surface fields in the medium above the scattering dielectric surface. It should be noted that these expressions are exact. However, they cannot in general be solved analytically and therefore approximations have to be introduced. Below we will show that by introducing an approximation called the *tangent plane approximation* (or the *Kirchhoff approximation*), closed analytical solutions can be obtained to the scattering problem.

3.2 The tangent plane approximation and the Kirchhoff fields

In the Kirchhoff approach, the total fields at any point of the surface (i.e., the incident plus the scattered one, to be considered inside the integral (3.1.1)) are approximated by the fields that would be present on an infinitely extended tangent plane at that particular point on the surface. The reflection is therefore considered to be locally specular. It is due to this fact that the Kirchhoff approximation is also referred to as the *tangent plane approximation*. The Kirchhoff approach requires to be valid that every point on the surface has a large radius of curvature relative to the wavelength of the incident field.

Thus, under the tangent-plane approximation, the total field at a point on the surface is assumed equal to the incident field plus the field reflected by an infinite plane tangent to the point. Hence, the tangential surface fields are (Ulaby et al, 1982):

$$\hat{\mathbf{n}}\times\mathbf{E}=\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_k=\hat{\mathbf{n}}\times\left(\mathbf{E}^i+\mathbf{E}^r\right) \tag{3.2.1}$$

$$\hat{\mathbf{n}}\times\mathbf{H}=\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_k=\hat{\mathbf{n}}\times\left(\mathbf{H}^i+\mathbf{H}^r\right) \tag{3.2.2}$$

Here the subscript *k* stands for the Kirchhoff approximation.
The way to proceed from here, in most presentations of the Kirchhoff method, consists in expressing the tangential fields under the Kirchhoff approximation in terms of the incident electric field components and the local Fresnel reflection coefficients, which depend on the local angles of incidence. This results in the following expressions:

$$\hat{\mathbf{n}}\times\mathbf{E}=\left[\ \left(1+R_h\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{t}}\right)\left(\hat{\mathbf{n}}\times\hat{\mathbf{t}}\right)-\left(1-R_v\right)\left(\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}_i\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{d}}\right)\hat{\mathbf{t}}\ \right]E_0\,e^{-jk\hat{\mathbf{k}}_i\cdot\mathbf{r}} \tag{3.2.3}$$

$$\eta\left(\hat{\mathbf{n}}\times\mathbf{H}\right)=-\left[\ \left(1-R_h\right)\left(\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}_i\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{t}}\right)\hat{\mathbf{t}}+\left(1+R_v\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{d}}\right)\left(\hat{\mathbf{n}}\times\hat{\mathbf{t}}\right)\right]E_0\,e^{-jk\hat{\mathbf{k}}_i\cdot\mathbf{r}} \tag{3.2.4}$$

where the unit vectors $\hat{\mathbf{t}}$, $\hat{\mathbf{d}}$, $\hat{\mathbf{k}}_i$ define the local reference coordinate system (see (Fung, 1994)) and $\hat{\mathbf{n}}$ is the unit normal vector to the interface in the above medium. R_v and R_h are the Fresnel reflection coefficients for vertical and horizontal polarisation respectively.
Upon substituting (3.2.3) and (3.2.4) in (3.1.1), the scattered field is:

$$E^s_{qp}=K\int\left\{\hat{\mathbf{q}}\cdot\left[\hat{\mathbf{k}}_s\times\left(\hat{\mathbf{n}}\times\mathbf{E}_p\right)+\eta\left(\hat{\mathbf{n}}\times\mathbf{H}_p\right)\right]\right\}e^{j(\mathbf{k}_s-\mathbf{k}_i)\cdot\mathbf{r}}ds \tag{3.2.5}$$

where the phase factor, $\exp\left(-jk\hat{\mathbf{k}}_i\cdot\mathbf{r}\right)$, of the incident wave has been pointed out from the equations (3.2.3) and (3.2.4). Such equation represents the scattered field formulated under the tangent-plane, or Kirchhoff approximation. As it stands the expression is a complicated function of the surface function and its partial derivatives. No analytic solution has been obtained from (3.2.5) without additional simplifying assumptions. Here we will show the results presented in (Ulaby et al, 1982): for surface with large (with respect to wavelength) standard deviation of surface heights, for which the stationary-phase approximation (Geometric Optics, GO) will be used, and for surfaces with small slopes and a medium or small standard deviation of surface heights, for which a scalar approximation (Physical Optics, PO) will be used.

3.2.1 The scattered field under the stationary-phase approximation (Geometric Optic, GO)

Under the stationary-phase approximation the local tangent plane on a surface point can be considered infinitely wide and, as consequence, the angular re-irradiation pattern originating from that specific point can be represented by a delta function centred in the specular direction. This means that scattering can occur only along directions for which there are specular points on the surface. Hence local diffraction effects are excluded. The approximating relations are obtained from the phase *Q* of (3.2.5), that is:

$$Q=k\left(\hat{\mathbf{k}}_s-\hat{\mathbf{k}}_i\right)\cdot\mathbf{r}\equiv\mathbf{q}\cdot\mathbf{r}=q_x x+q_y y+q_z z \tag{3.2.1.1}$$

where

$$\hat{\mathbf{k}}_s = \hat{\mathbf{x}}\sin\theta_s\cos\phi_s + \hat{\mathbf{y}}\sin\theta_s\sin\phi_s + \hat{\mathbf{z}}\cos\theta_s \tag{3.2.1.2}$$

$$\hat{\mathbf{k}}_i = \hat{\mathbf{x}}\sin\theta_i\cos\phi_i + \hat{\mathbf{y}}\sin\theta_i\sin\phi_i - \hat{\mathbf{z}}\cos\theta_i \tag{3.2.1.3}$$

$$q_x = k\left(\sin\theta_s\cos\phi_s - \sin\theta_i\cos\phi_i\right) \tag{3.2.1.4}$$

$$q_y = k\left(\sin\theta_s\sin\phi_s - \sin\theta_i\sin\phi_i\right) \tag{3.2.1.5}$$

$$q_z = k\left(\cos\theta_s + \cos\theta_i\right) \tag{3.2.1.6}$$

The phase Q is said to be stationary at a point if its rate of change is zero at the point, that is:

$$\frac{\partial Q}{\partial x} = 0 = q_x + q_z\frac{\partial z}{\partial x}$$

$$\frac{\partial Q}{\partial y} = 0 = q_y + q_z\frac{\partial z}{\partial y}$$

Hence, the partial derivatives of the surface slopes can be replaced by the components of the phase as:

$$Z_x = \frac{\partial z}{\partial x} = -\frac{q_x}{q_z} \tag{3.2.1.7}$$

$$Z_y = \frac{\partial z}{\partial y} = -\frac{q_y}{q_z} \tag{3.2.1.8}$$

Since, the local unit vector $\hat{\mathbf{n}}$ is a function of the surface derivatives:

$$\hat{\mathbf{n}} = \frac{-Z_x\hat{\mathbf{x}} - Z_y\hat{\mathbf{y}} + \hat{\mathbf{z}}}{\sqrt{1+Z_x^2+Z_y^2}} \tag{3.1.1.9}$$

the use of (3.2.1.7) and (3.2.1.8) makes $\hat{\mathbf{n}}\times\mathbf{E}$ and $\hat{\mathbf{n}}\times\mathbf{H}$ independent on the integration variables. Thus, the expression for $\mathbf{E}^s$ can be rewritten as:

$$\mathbf{E}^s = K\hat{\mathbf{k}}_s\times\left[\left(\hat{\mathbf{n}}\times\mathbf{E}\right) - \eta\hat{\mathbf{k}}_s\times\left(\hat{\mathbf{n}}\times\mathbf{H}\right)\right] I_1 \tag{3.2.1.10}$$

where

$$I_1 = \int e^{jk\left(\hat{\mathbf{k}}_s - \hat{\mathbf{k}}_i\right)\cdot\mathbf{r}} ds \tag{3.2.1.11}$$

The scattering field corresponding to transmission of $\hat{\mathbf{p}}$ polarisation and reception of $\hat{\mathbf{q}}$ polarisation can be written as (Ulaby et al, 1982):

$$E_{qp}^{s} = \hat{\mathbf{q}} \cdot \mathbf{E}^{s} \equiv K I_1 E_0 U_{qp} \tag{3.2.1.12}$$

where

$$U_{qp} = \frac{1}{E_0} \hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_s \times \left[\left(\hat{\mathbf{n}} \times \mathbf{E} \right) - \eta \hat{\mathbf{k}}_s \times \left(\hat{\mathbf{n}} \times \mathbf{H} \right) \right] \tag{3.2.1.13}$$

To compute the scattering coefficient, defined in (2.4.1), for different polarisation states, it is necessary to calculate the ensemble average of $\left| I_1 \right|^2$:

$$\left\langle \left| I_1 \right|^2 \right\rangle = \iint \left\langle e^{jk\left(\hat{\mathbf{k}}_s - \hat{\mathbf{k}}_i\right) \cdot \left(\mathbf{r} - \mathbf{r}'\right)} \right\rangle ds ds' \tag{3.2.1.14}$$

By assuming the surface roughness as a stationary and isotropic Gaussian random process, with zero mean, variance σ^2, and correlation coefficient ρ, and in the assumption that the standard deviation of surface heights is large (that is, $\left(q_z \sigma\right)^2$ large) the integral can be solved. The result is (Ulaby et al, 1982):

$$\left\langle \left| I_1 \right|^2 \right\rangle = \frac{2\pi A_0 q^2}{q_z^4 \sigma^2 \left| \rho''(0) \right|} \exp\left[-\frac{q_x^2 + q_y^2}{2 q_z^2 \sigma^2 \left| \rho''(0) \right|} \right] \tag{3.2.1.15}$$

where the illuminated area A_0 is $\left(2L\right)^2$, $\rho''(0)$ is the second derivatives of ρ evaluated at the origin and $\sigma^2 \left| \rho''(0) \right|$ corresponds to the mean-squared slope of the surface (Ulaby et al, 1982) (Section 2.3.1).
Upon substituting (3.2.1.15) into the product in the scattered-field expression, it follows:

$$\left\langle E_{qp}^{s} E_{qp}^{s\,*} \right\rangle = \left| K E_0 U_{qp} \right|^2 \left\langle \left| I_1 \right|^2 \right\rangle \tag{3.2.1.16}$$

Substituting (3.2.1.16) in the definition of the scattering coefficient given by equation (2.4.1), it assumes the following expression:

$$\sigma_{qp}^{o} = \frac{\left(kq \left| U_{qp} \right|^2 \right)}{2 q_z^4 \sigma^2 \left| \rho''(0) \right|} \exp\left[-\frac{q_x^2 + q_y^2}{2 q_z^2 \sigma^2 \left| \rho''(0) \right|} \right] \tag{3.2.1.17}$$

In the derivation of σ_{qp}^{o}, the effects of shadowing and multiple scattering have been ignored. It is important to underline that (3.2.1.17) is valid only for surface with sufficiently large standard deviation of surface heights. Under such assumption, that is $\left(q_z \sigma\right)^2$ large, the scattering is purely incoherent. As $\left(q_z \sigma\right)^2$ decreases, some scattered energy begins to appear in the coherent component. To examine such situation, a different approximation to the tangential fields is needed to permit small $\left(q_z \sigma\right)^2$. This is discussed in the next section.

3.2.2 The scattered field under the scalar approximation (Physical Optics, PO)

A different Kirchhoff approach is the Physical Optics solution to (3.1.1). The Physical Optics approach involves the integration of the Kirchhoff scattered field over the entire rough surface, not just the portions of surface which contribute specularly to the scattered direction. Unlike the Geometric Optics solution, the Physical Optics solution predicts a coherent component.

The power in the incoherent reflected field can be found by expanding the Stratton-Chu equation in a Taylor series in surface slope distribution. In (Ulaby et al, 1982) the Physical Optics solution is called scalar approximation because slopes are ignored in the surface coordinate system, leading to a decoupling of polarisation in the vector scattering equations. Accordingly, the basic scattered-field expression can be rewritten in the form:

$$E_{qp}^{s} = KE_0 \int \overline{U}_{qp} \exp\left[jk\left(\hat{\mathbf{k}}_s - \hat{\mathbf{k}}_i\right)\cdot \mathbf{r} \right] ds \tag{3.2.2.1}$$

where $\overline{U}_{qp}$ are given in (Ulaby et al, 1982). To find $\left\langle E_{qp}^{s} E_{qp}^{s\,*} \right\rangle$ for the scattering-coefficient computation, the following integral needs to be computed:

$$I = \int\int \left\langle \overline{U}_{qp} \overline{U}_{qp}^{*} \exp\left[jk\left(\hat{\mathbf{k}}_s - \hat{\mathbf{k}}_i\right)\cdot\left(\mathbf{r} - \mathbf{r}'\right) \right] \right\rangle ds ds' \tag{3.2.2.2}$$

Since all $\overline{U}_{qp}$ are expressed in a Taylor series in surface slope distribution, Z_x and Z_y:

$$\overline{U}_{qp} = a_0 + a_1 Z_x + a_2 Z_y \tag{3.2.2.3}$$

where a_i are polarisation-dependent coefficients, the product $\overline{U}_{qp}\overline{U}_{qp}^{*}$ can be written up to the first order in slope as:

$$\overline{U}_{qp}\overline{U}_{qp}^{*} \approx a_0 a_0^{*} + a_0 a_1^{*} Z_x + a_0^{*} a_1 Z_x + a_0 a_2^{*} Z_y + a_0^{*} a_2 Z_y \tag{3.2.2.4}$$

Since $\left(q_z\sigma\right)^2$ is no longer required to be large and assuming the size of the illuminated area equal to $2L \times 2L$, the ensemble average of the first term in (3.2.2.4) can be expressed as (for more details see (Ulaby et al, 1982))

$$I_0 = \left|a_0\right|^2 e^{-q_z^2\sigma^2} \sum_{n=0}^{\infty} \frac{\left(q_z^2\sigma^2\right)^n}{n!} \int_{-2L}^{2L}\int_{-2L}^{2L} \rho^n\left(2L - |u|\right)\left(2L - |v|\right) e^{jq_x u + jq_y v} du dv \tag{3.2.2.5}$$

where the $n = 0$ term corresponds to coherent scattering. It can be shown that this coherent-scattering coefficient can be expressed as:

$$\sigma_{qp}^{o\,c} = \pi k^2 \left|a_0\right|^2 \delta\left(q_x\right)\delta\left(q_y\right) e^{-q_z^2\sigma^2} \tag{3.2.2.6}$$

which shows that coherent scattering is important only when $q_z\sigma$ is small. The rest of the series in (3.2.2.5) represents incoherent scattering. The integral I_0 for $n \geq 1$ can be rewritten in the following manner pointing out the illuminated area $A_0 = (2L)^2$:

$$I_0 = |a_0|^2 e^{-q_z^2\sigma^2} A_0 \sum_{n=1}^{\infty} \frac{\left(q_z^2\sigma^2\right)^n}{n!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^n e^{jq_x u + jq_y v} du dv \tag{3.2.2.7}$$

For an isotropically rough surface with correlation length l and Gaussian normalised autocorrelation function, $\rho = \exp\left[-\xi^2/l^2\right]$, the integral (3.2.2.7) can be shown to be:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-n\xi^2/l^2 + jq_x u + jq_y v} du dv = \frac{\pi l^2}{n} e^{-\frac{\left(q_x^2+q_y^2\right)l^2}{4n}} \tag{3.1.2.8}$$

It is clear that different solutions may be obtained for the integral if the normalised surface autocorrelation function is assumed to take some other functional forms. Upon substituting (3.2.2.7) and (3.2.2.8) into the factor $\left\langle E_{qp}^s E_{qp}^{s\,*} \right\rangle$, the scattering coefficient for the incoherent part of the $|a_0|^2$ term has the following expression:

$$\sigma_{qp}^{o\ inc} = \left(|a_0| kl/2\right)^2 e^{-q_z^2\sigma^2} \sum_{n=1}^{\infty} \frac{\left(q_z^2\sigma^2\right)^n}{n!n} e^{-\frac{\left(q_x^2+q_y^2\right)l^2}{4n}} \tag{3.2.2.9}$$

If the normalised surface autocorrelation is not known, $\sigma_{qp}^{o\ inc}$ can be written as:

$$\sigma_{qp}^{o\ inc} = \frac{k^2|a_0|^2}{4\pi} e^{-q_z^2\sigma^2} \sum_{n=1}^{\infty} \frac{\left(q_z^2\sigma^2\right)^n}{n!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^n e^{jq_x u + jq_y v} du dv \tag{3.2.2.10}$$

An additional contribution to the total scattering coefficient comes from the slope terms in (3.2.2.4). It can be computed taking into account in the ensemble average $\left\langle E_{qp}^s E_{qp}^{s\,*} \right\rangle$ the integrals of the slope terms in the *x*- and *y*-direction. The results of such integrals for a Gaussian normalised autocorrelation function are reported in (Ulaby et al, 1982). Also the expressions of the polarisation-dependent coefficients a_i can be found in the same reference. However, the expressions of the coefficient a_0 for each polarisation are reported below for the two particular cases of backscattering and scattering in the specular direction.

In the backscattering:

HH polarisation: $a_0 = 2R_h(\theta_i)\cos\theta_i$

VH polarisation: $a_0 = 0$

VV polarisation: $a_0 = -2R_v(\theta_i)\cos\theta_i$

HV polarisation: $a_0 = 0$

Conversely, in the specular direction case:

HH polarisation: $a_0 = -2R_h(\theta_i)\cos\theta_i$

VH polarisation: $a_0 = 0$

VV polarisation: $a_0 = 2R_v(\theta_i)\cos\theta_i$

HV polarisation: $a_0 = 0$

The quantity $q_{x,y,z}$ are defined in the previous section.

3.3 On the range of validity of the Kirchhoff method and shadowing effects

The basic assumption of the Kirchhoff method is that plane-boundary reflection occurs at every point on the surface. Thus, when statistical surfaces are considered, their horizontal-scale roughness, the correlation length l, must be larger than the electromagnetic wavelength, while their vertical-scale roughness, the standard deviation σ of surface heights, must be small enough so that the average radius of curvature is larger than the electromagnetic wavelength. Mathematically, for stationary isotropic Gaussian surface the above-stated restriction are (Ulaby et al, 1982):

$$kl > 6 \tag{3.3.1}$$

$$\sigma < \frac{l^2}{2.76\lambda} \tag{3.3.2}$$

where k is the wave number and λ is the electromagnetic wavelength. Note that the surface standard deviation should be small relative to the correlation length, but it can be comparable to or even larger than the electromagnetic wavelength. This means that large standard deviations can be tolerated if the correlation length is large enough to preserve an acceptable average radius of curvature. The conditions reported above are for the Kirchhoff approximation. The scattering models described in section 3.2.1 and 3.2.2 require additional approximations reported in the following table:

Validity limits of Kirchhoff Approximation (KA) (Gaussian surface)		
$l^2 > 2.76\sigma\lambda$	and	$kl > 6$
Stationary Phase Aproximation (GO)		Scalar Approximation (PO)
$k\sigma > 2$		$k\sigma < 1$ and $rms_{slope} < 0.25$

Table 3.3.1. Validity of GO and PO for stationary isotropic Gaussian surfaces with standard deviation σ and correlation length l.

3.4 Some concluding remarks on the Kirchhoff method

As was mentioned in the previous paragraph, the Kirchhoff method does neither in itself account for shadowing and nor does it (in the form described here) account for multiple scattering on the surface. Due to the lack of these two effects energy conservation is not satisfied. However, in (Ulaby et al, 1982) this conservation is demonstrating with the inclusion of these two effects.

In the literature, the surface height distribution is in most cases assumed to be Gaussian. The reason for this is, as mentioned previously, that the surface roughness rms height and the autocorrelation function entirely determine the random process, and therefore the bistatic scattering coefficient can be expressed in terms of these two quantities.

The Kirchhoff method has been applied to surfaces described by fractal geometry. As an example we can mention that in (Franceschetti et al, 1999) a fractional Brownian motion model was used for modelling the scattering from natural rough surface. In combination with the Kirchhoff method an analytical solution for the bistatic scattering coefficient was obtained.

4. The small perturbation method

The Small Perturbation Method (SPM) belongs to a large family of perturbation expansion solutions to the wave equation. The approach is based on formulating the scattering as a partial differential equation boundary value problem. The basic idea is to find a solution in terms of plane waves that matches the surface boundary conditions, which state that the tangential component of the field must be continuous across the boundary. The surface fields are expanded in a perturbation series with respect to surface height, e.g., $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 + \ldots$. In the expansion $\mathbf{E}_0$ would be the surface field if the surface was flat. The philosophy behind this approach is that small effective surface currents on a mean surface replace the role of a small-scale roughness. So this method applies to surfaces with small surface height variations and small surface slopes compared with the wavelength but independently of the radius of curvature of the surface. Therefore, the surface needs no longer to be approximated by planes. The small-scale roughness is expanded in a Fourier series and the contribution to the field is therefore analysed in terms of different wavelength components.
Here we will report only the expressions of the bistatic scattering coefficient. A more detailed description of their computation process can be found in (Ulaby et al, 1982).

4.1 A small presentation of the SPM

The zero order solution of the SPM is the same as for a plane interface, while the first order solution gives the incoherent scattered field due to single scattering. For the latter case, the bistatic scattering coefficient for either a horizontally or vertically polarised incident wave is (Ulaby et al, 1982):

$$\sigma_{qp}^{o} = 8\left|k^2\sigma\cos\theta_i\cos\theta_s\alpha_{qp}\right|^2 W\left(k_x + k\sin\theta_i, k_y\right) \tag{4.1.1}$$

where

$$k_x = -k\sin\theta_s\cos\phi_s$$

$$k_y = -k\sin\theta_s\sin\phi_s$$

$$W\left(k_x, k_y\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(u,v)e^{-jk_x u - jk_y v}dudv$$

σ and $\rho(u,v)$ are, respectively, the variance of surface heights and the surface correlation coefficient; α_{qp} are coefficients that depend on polarisation, incidence and scattering angle, and on complex relative dielectric constant ε_c of the homogeneous medium below the interface. The detailed expressions of α_{qp} are reported in (Ulaby et al, 1982).

4.2 Some remarks on the region of validity of the SPM

The Small Perturbation Method is applied to surfaces with a surface height standard deviation much less than the incident wavelength (5 percent or less) and an average surface slope comparable to or less than the surface standard deviation times the wave number. For a surface with Gaussian correlation function, such two conditions can be expressed

analytically as follows, but they should be viewed only as a guideline for applying the SPM scattering model:

$$k\sigma < 0.3$$

$$\sqrt{2}\,\sigma/l < 0.3$$

The SPM has been compared to more accurate numerical simulations in (Thorsos & Jackson, 1989; 1991) for one-dimensional rough surfaces with a Gaussian roughness spectrum. Under these conditions the authors show that the first-order SPM gives accurate results for $k\sigma << 1$ and $kl \approx 1$. The results also show that for $k\sigma << 1$ and $kl > 6$, the sum of the first three orders of the SPM is required to obtain accurate results.
It has been argued that the SPM does account for multiple scattering up to the order of the perturbative expansion. This means that the first order perturbative solution does not account for multiple scattering but that some multiple scattering effects can be observed in the higher order solutions.

Validity limits of Small Perturbation Method (SPM) (Gaussian surface)		
$k\sigma < 0.3$	and	$rms_{slope} < 0.3$

Table 4.2.1. Validity of SPM for stationary isotropic Gaussian surfaces with standard deviation σ and root mean square slope rms_{slope}.

5. The Integral Equation Method (IEM)

A relatively new method for calculating scattering of electromagnetic waves from rough surfaces is the Integral Equation Method (IEM). The IEM has been used extensively in the microwave region in recent years and it has proved to provide good predictions for a wide range of surface profiles. The method can be viewed as an extension of the Kirchhoff method and the Small Perturbation Method since it has been shown to reproduce results of these two methods in appropriate limits. The IEM is a relatively complicated method in its general form (including multiple scattering) and it is beyond the scope of the present overview to give a full presentation of the method. A more detailed presentation of the IEM can be found in (Fung, 1994).

5.1 On the formulation of the IEM

The starting point of the IEM is the Stratton-Chu integral for the scattered field, equation (3.1.1). The tangential surface fields which enter the Stratton-Chu integral are given in equations (2.2.11) - (2.2.12) and (2.2.15) - (2.2.16). In the Kirchhoff approach, the tangential fields are approximated using the tangent plane approximation, replacing the complete tangential surface fields with the Kirchhoff tangential surface fields of equations (3.2.1) and (3.2.2). It is clear that the Kirchhoff tangential surface fields cannot provide alone a good estimate of the surface fields since the integral form in equations (2.2.11) - (2.2.12) are not accounted for in the Kirchhoff approach. In the IEM, a complementary term is included in equations (3.2.1) and (3.2.2) to correct for this:

$$\hat{\mathbf{n}}\times\mathbf{E}=\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_k+\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_c \tag{5.1.1}$$

$$\hat{\mathbf{n}}\times\mathbf{H}=\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_k+\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_c \tag{5.1.2}$$

In these equations, the first terms on the right hand side are the tangential fields under Kirchhoff approximation and the complementary fields are given by:

$$\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_c=\hat{\mathbf{n}}\times\left(\mathbf{E}^i-\mathbf{E}^r\right)-\frac{2}{4\pi}\hat{\mathbf{n}}\times\int\varepsilon ds' \tag{5.1.3}$$

$$\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_c=\hat{\mathbf{n}}\times\left(\mathbf{H}^i-\mathbf{H}^r\right)+\frac{2}{4\pi}\hat{\mathbf{n}}\times\int\boldsymbol{\mathcal{H}}\,ds' \tag{5.1.4}$$

$\mathbf{E}^r$ and $\mathbf{H}^r$ being the reflected electric and magnetic fields propagating along the reflected direction. To use (5.1.1) and (5.1.2) for estimating the tangential field, both the Kirchhoff field and the complementary field need to be expressed in terms of the incident field components and the surface reflectivity properties. Using the local coordinate system defined by the unit vectors $\hat{\mathbf{t}}$, $\hat{\mathbf{d}}$, $\hat{\mathbf{k}}_i$ (for their expressions refer to (Fung, 1994)), the incident electric and magnetic field can be expressed into locally horizontally and vertically polarised components. Accordingly, after some manipulations (see (Fung, 1994) for more details), the Kirchhoff and complementary tangential fields can be rewritten as:

$$\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_k=\hat{\mathbf{n}}\times\left[\left(1+R_h\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{t}}\right)\hat{\mathbf{t}}+\left(1-R_v\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{d}}\right)\hat{\mathbf{d}}\right]E^i \tag{5.1.5}$$

$$\eta\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_k=\hat{\mathbf{n}}\times\left[\left(1-R_h\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{t}}\right)\hat{\mathbf{d}}+\left(1+R_v\right)\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{d}}\right)\hat{\mathbf{t}}\right]E^i \tag{5.1.6}$$

$$\begin{aligned}\left(\hat{\mathbf{n}}\times\mathbf{E}\right)_c=&-\frac{1}{4\pi}\left(\hat{\mathbf{n}}\times\hat{\mathbf{t}}\right)\left\{\hat{\mathbf{n}}\times\hat{\mathbf{t}}\cdot\hat{\mathbf{n}}\times\int\left[\left(1+R_h\right)\varepsilon+\left(1-R_h\right)\varepsilon_t\right]ds'\right\}\\&-\frac{1}{4\pi}\hat{\mathbf{t}}\left\{\hat{\mathbf{t}}\cdot\hat{\mathbf{n}}\times\int\left[\left(1-R_v\right)\varepsilon+\left(1+R_v\right)\varepsilon_t\right]ds'\right\}\end{aligned} \tag{5.1.7}$$

$$\begin{aligned}\left(\hat{\mathbf{n}}\times\mathbf{H}\right)_c=&\frac{1}{4\pi}\left(\hat{\mathbf{n}}\times\hat{\mathbf{t}}\right)\left\{\hat{\mathbf{n}}\times\hat{\mathbf{t}}\cdot\hat{\mathbf{n}}\times\int\left[\left(1+R_v\right)\boldsymbol{\mathcal{H}}+\left(1-R_v\right)\boldsymbol{\mathcal{H}}_t\right]ds'\right\}\\&+\frac{1}{4\pi}\hat{\mathbf{t}}\left\{\hat{\mathbf{t}}\cdot\hat{\mathbf{n}}\times\int\left[\left(1-R_h\right)\boldsymbol{\mathcal{H}}+\left(1+R_h\right)\boldsymbol{\mathcal{H}}_t\right]ds'\right\}\end{aligned} \tag{5.1.8}$$

It can be noted that, while (5.1.5) and (5.1.6) are expressed in terms of known quantities, that is the incident electric or magnetic fields, the local Fresnel reflection coefficient and the local incident angle, (5.1.7) and (5.1.8) are integral equations. In order to obtain estimates of (5.1.7) and (5.1.8), IEM substitutes the unknown expressions of the tangential fields in the right-hand side of (5.1.7) and (5.1.8), that is the $\left(\hat{\mathbf{n}}'\times\mathbf{E}'\right)$ and $\left(\hat{\mathbf{n}}'\times\mathbf{H}'\right)$ terms which appear in ε, ε_t, $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{H}}_t$, with the Kirchhoff tangential fields, $\left(\hat{\mathbf{n}}'\times\mathbf{E}'\right)_k$ and $\left(\hat{\mathbf{n}}'\times\mathbf{H}'\right)_k$, respectively.

This is the fundamental approximation adopted by IEM model. However, even with this simplification the obtained integral expressions remain too complex for practical use.

Much simpler approximate expressions of the tangential Kirchhoff and complementary fields can be obtained differentiating them for each linear incident and scattered polarisation. The resulting approximated equations (electric and magnetic surface field equations for horizontal, vertical and cross polarisation) can be found in (Fung, 1994).
Then, the simplified tangential surface fields can be inserted in the Stratton-Chu integral. The far field scattered from the rough surface can be expressed as a combination of the Kirchhoff and the complementary term:

$$E_{qp}^{s} = E_{qp}^{k} + E_{qp}^{c} \tag{5.1.9}$$

where

$$E_{qp}^{k} = CE_0 \int f_{qp}\, e^{j(\mathbf{k}_s - \mathbf{k}_i)\cdot\mathbf{r}} dxdy \tag{5.1.10}$$

and

$$E_{qp}^{c} = \frac{C}{8\pi^2}\int \tilde{F}_{qp} e^{j\mathbf{k}_s\cdot\mathbf{r}} dxdy = \frac{CE_0}{8\pi^2}\int F_{qp}\, e^{ju(x-x')+jv(y-y')+j\mathbf{k}_s\cdot\mathbf{r}-j\mathbf{k}_i\cdot\mathbf{r}'} dxdydudvdx'dy' \tag{5.1.11}$$

The quantities f_{qp} and $\tilde{F}_{qp}$, respectively the Kirchhoff and complementary field coefficients, that appear in the above equations are defined as follows:

$$f_{qp} = \left[\hat{\mathbf{q}}\times\mathbf{k}_s\cdot\left(\hat{\mathbf{n}}\times\mathbf{E}_p\right)_k + \eta\,\hat{\mathbf{q}}\cdot\left(\hat{\mathbf{n}}\times\mathbf{H}_p\right)_k\right] D_1 \Big/ E_i \tag{5.1.12}$$

$$\tilde{F}_{qp} = 8\pi^2\left[\hat{\mathbf{q}}\times\mathbf{k}_s\cdot\left(\hat{\mathbf{n}}\times\mathbf{E}_p\right)_c + \eta\,\hat{\mathbf{q}}\cdot\left(\hat{\mathbf{n}}\times\mathbf{H}_p\right)_c\right] D_1 \tag{5.1.13}$$

where $D_1 = \sqrt{1+Z_x^2+Z_y^2}$ and E^i is the complex amplitude of the incident electric field.
In general, both f_{qp} and $\tilde{F}_{qp}$ are dimensionless, complicated expressions and depended on spatial variables. Therefore several approximations are made to make these functions independent of spatial variables (Fung, 1994).
In particular, the f_{qp} coefficients depend on the Fresnel reflection coefficients, and hence on the local angle, and on the slope terms, Z_x and Z_y. The first dependency is removed by approximating the local incidence angle in the Fresnel reflection coefficients by the *incident angle*, θ_i, for surface with *small scale roughness* and by the *specular angle*, θ_{sp}, $\cos\theta_{sp} = -\hat{\mathbf{n}}\times\hat{\mathbf{k}}_i$, for surface with *large scale roughness*. The rule that defines the bound between the two regions is reported here assuming a Gaussian autocorrelation function:

$$R_{v,h} = \begin{cases} R_{v,h}(\theta_i) & k^2\sigma\, l < 1.2\sqrt{\varepsilon_r} \\ R_{v,h}(\theta_{spec}) & kl > 5 \end{cases} \tag{5.1.14}$$

In order to remove the dependence on the slope terms, the integral (5.1.10) is solved by parts and the edge terms were discarded.
To obtain the expressions of the complementary coefficients $\tilde{F}_{qp}$, the computation is rather lengthy and complicated. When the equations (2.2.15) - (2.2.18) are substituted in the

approximated expressions of tangential complementary fields, the spectral representations of Green's function and of its gradient are introduced, *assuming however the same Green's functions for both the medium*:

$$G = -\frac{1}{2\pi}\int \frac{j}{q} e^{ju(x-x')+jv(y-y')-jq|z-z'|} du dv \tag{5.1.15}$$

$$\nabla' G = -\frac{1}{2\pi}\int \frac{\mathbf{g}}{q} e^{ju(x-x')+jv(y-y')-jq|z-z'|} du dv \tag{5.1.16}$$

$\mathbf{g} = u\hat{\mathbf{x}} + v\hat{\mathbf{y}} \mp q\hat{\mathbf{z}}$ and $q = \sqrt{k^2 - u^2 - v^2}$ are the propagation vector and its z-component of the generic plane wave that appears in the plane waves expansion of the field, whereas z and z' are the random variables representing the surface heights at different locations on the random surface. In (Fung, 1994), the $|z - z'|$ terms and the term with the $\mp$ are dropped in the equations (5.1.15) and (5.1.16) in order to simplify the calculation. However, in an improved version of the IEM (see (Chen et al, 2000)) these terms are kept in the analysis. In addition, as was the case for the Kirchhoff coefficients, f_{qp}, the dependence through the slope terms is removed by integrating by parts and discarding the edge terms. Instead, as regard the Fresnel reflection coefficients, the local angle is always replaced by incident angle (Fung, 1994; Wu et al, 2001).

Moreover, it is important to underline that the tangential and normal field components that appear in the expressions of the $\tilde{F}_{qp}$ coefficients through equations (2.2.15) - (2.2.18) can be approximated by the tangential Kirchhoff fields. The complimentary field coefficients F_{qp} that appear in the right term of the equation (5.1.11) are obtained from the definition of the $\tilde{F}_{qp}$ after the Green's function and its gradient are replaced by the *simplified* spectral representation, above mentioned, and after the phase factor of the Green function and u, v, x', y' integrations are factored out. The expressions of such coefficients together with the expressions of the Kirchhoff ones are reported in (Brogioni et al, 2010).

Once the field coefficients, f_{qp} and F_{qp}, are made independent of spatial variables, it is possible to provide the expression of the incoherent scattered power:

$$\begin{aligned}\left\langle \left|E_{qp}^s\right|^2 \right\rangle - \left|\left\langle E_{qp}^s \right\rangle\right|^2 &= \left\langle E_{qp}^s E_{qp}^{s\,*} \right\rangle - \left\langle E_{qp}^s \right\rangle \left\langle E_{qp}^s \right\rangle^* = \\ &= \left\langle E_{qp}^k E_{qp}^{k\,*} \right\rangle - \left\langle E_{qp}^k \right\rangle \left\langle E_{qp}^k \right\rangle^* + \\ &+ 2\,\mathrm{Re}\left[\left\langle E_{qp}^c E_{qp}^{k\,*} \right\rangle - \left\langle E_{qp}^c \right\rangle \left\langle E_{qp}^k \right\rangle^* \right] + \left\langle E_{qp}^c E_{qp}^{c\,*} \right\rangle - \left\langle E_{qp}^c \right\rangle \left\langle E_{qp}^c \right\rangle^*\end{aligned} \tag{5.1.17}$$

and from this the bistatic scattering coefficient:

$$\sigma_{qp}^o = \sigma_{qp}^k + \sigma_{qp}^{kc} + \sigma_{qp}^c \tag{5.1.18}$$

From the above expression it follows that the scattering coefficient is given by the sum of three terms: the Kirchhoff, the complementary and the cross term. The first is originated by Kirchhoff fields, the second by the interaction between Kirchhoff and complementary fields, whereas the last is due only to complementary fields.

To carry out the average operation an assumption about the type of surface height distribution is necessary. In order to simplify the calculation of the incoherent power terms the rough surface is assumed characterised by a Gaussian height distribution. Accordingly, the terms in (5.1.18) assume the following expressions, reported in (Fung, 1994):

$$\sigma_{qp}^{k} = \frac{k^2}{4\pi}\left|f_{qp}\right|^2 e^{-\sigma^2\left(k_{sz}+k_z\right)^2} \int\left\{\exp\left[\sigma^2\left(k_{sz}+k_z\right)^2\rho(\xi,\zeta)\right]-1\right\}\exp\left[j\left(k_{sx}-k_x\right)\xi + j\left(k_{sy}-k_y\right)\zeta\right]d\xi\, d\zeta \tag{5.1.19}$$

$$\begin{aligned}\sigma_{qp}^{kc} = \frac{k^2}{16\pi^3}\mathrm{Re}\Big\{&\int\left(F_{qp}f_{qp}^{\ *}\right)\exp\left[-\sigma^2\left(k_{sz}^2+k_z^2+k_zk_{sz}\right)\right] \\ &\iint\exp\left[-\sigma^2k_zk_{sz}\rho(\xi-\xi',\zeta-\zeta')\right]\Big\{\exp\left[\sigma^2k_{sz}\left(k_{sz}+k_z\right)\rho(\xi,\zeta)+\right. \\ &\left.\sigma^2k_z\left(k_{sz}+k_z\right)\rho(\xi',\zeta')\right]-1\Big\}\exp\left[jk_{sz}\xi+jk_{sy}\zeta+j\nu(\zeta-\zeta')+ju(\xi-\xi')\right] \\ &\exp\left[-jk_x\xi'-jk_y\zeta'\right]d\xi\, d\zeta\, d\xi'\, d\zeta'\, du dv\Big\}\end{aligned} \tag{5.1.20}$$

$$\begin{aligned}\sigma_{qp}^{c} = \left|\frac{k}{16\pi^{2.5}}\right|^2\Big\{&\iiint\left(F_{qp}F_{qp}^{\ *}\right)\exp\left[-\sigma^2\left(k_{sz}^2+k_z^2\right)\right] \\ &\exp\left[-\sigma^2k_zk_{sz}\rho(\tau+\xi-\xi',\kappa+\zeta-\zeta')\right]\left(\exp\left[\sigma^2k_{sz}^2\rho(\xi,\zeta)\right]\right. \\ &\left.\exp\left\{\sigma^2k_z\rho(\xi',\zeta')+\sigma^2k_zk_{sz}\left[\rho(\xi+\tau,\zeta+\kappa)+\rho(\xi'-\tau,\zeta'-\kappa)\right]\right\}-1\right) \\ &\exp\left\{j\left[\left(k_{sx}+u\right)\xi+\left(k_{sy}+v\right)\zeta-\left(k_x+u\right)\xi'-\left(k_y+v\right)\zeta'\right]\right\} \\ &\exp\left\{j\left[\left(u-u'\right)\tau+\left(v-v'\right)\kappa\right]\right\}d\xi\, d\zeta\, d\xi'\, d\zeta'\, d\tau\, d\kappa\, du\, dv\, du'dv'\Big\}\end{aligned} \tag{5.1.21}$$

The above expressions consist of multiple integrals which are too complex and hence not practical to use. In order to evaluate these integrals, the model is approximated in two different forms depending upon whether the surface height is moderate or large in terms of the incident wavelength ($k\sigma$). The first case is referred to as low frequency approximation, whilst the other is referred to as high frequency approximation. An indicative threshold value of $k\sigma < 2$ is reported in (Fung, 1994). The detailed expressions of σ_{qp}^{k}, σ_{qp}^{kc}, σ_{qp}^{c} valid separately when $k\sigma < 2$ and for large $k\sigma$ are given in (Fung, 1994) and are not reported here. For both the approximations, in the expression of the bistatic scattering coefficient two types of terms can be distinguished: one representing single-scattering and the other representing multiple-scattering. The latter may be viewed as a correction to the single term for both the high- and the low-frequency regions. This division is important to identify weather single or multiple scattering is significant for applications. For completeness we report here the total single scattering coefficient obtained by selecting the single scattering contributions in the expressions of σ_{qp}^{k}, σ_{qp}^{kc}, σ_{qp}^{c} valid when $k\sigma < 2$ (for the detailed explanation refer to (Fung, 1994)):

$$\sigma^{o}_{qp} = \frac{k^2}{2} e^{-\sigma^2 (k_z^2 + k_{sz}^2)} \sum_{n=1}^{\infty} \frac{\sigma^{2n}}{n!} \left| (k_z + k_{sz})^n \; f_{qp} \; e^{-\sigma^2 k_z k_{sz}} + \frac{(k_{sz})^n F_{qp}(-k_x, -k_y) + (k_z)^n F_{qp}(-k_{sx}, -k_{sy})}{2} \right|^2 W^{(n)}(k_{sx} - k_x, k_{sy} - k_y)$$

(5.1.22)

6. Conclusions

We have presented the results from a literature search of models for scattering of electromagnetic waves from random rough surfaces. In particular we have focused on the calculation of the *bistatic scattering coefficient* in three different classes of methods: the *Kirchhoff Approximation*, the *Method of Small Perturbation* and the *Integral Equation Method.* Of these, the first two, are amongst the early approaches which however are still much used. The latter is an example of more recent approaches which have been developed as an attempt to extend the validity of the former methods.

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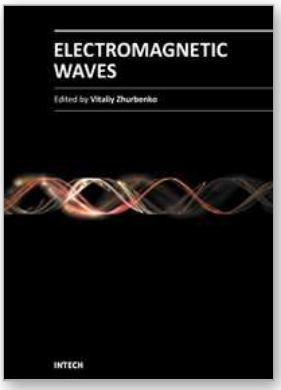

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